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THE OPERATOR EQUATION $AXB = X$ AND THE FUGLEDE–PUTNAM TYPE PROPERTY

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Abstract. In this paper, we study some connections between solutions A and B satisfying the operator equation $AXB = X$. We also investigate several properties between such solutions A and B. In particular, we show that if A has the single valued extension property, then so does B when X is injective. Moreover, we consider the (weak) Fuglede-Putnam type property (defined below) and investigate the local spectral properties between the solutions A and B under the Fuglede-Putnam type property.

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