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COMPLEX WEYL SYMBOLS OF THE EXTENDED METAPLECTIC REPRESENTATION OPERATORS

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Abstract. We consider the extended metaplectic representation of the semi-direct product of the Heisenberg group and the symplectic group (the Jacobi group). We give explicit formulas for the Berezin symbols and for the complex Weyl symbols of the corresponding representation operators. Then we deduce formulas for the symbols of the representation operators in the classical Weyl calculus. As an application, we find the classical Weyl symbol of the exponential of an operator whose Weyl symbol is a polynomial on $\mathbb{R}^{2n}$ of degree $\leqslant 2$, recovering a result of L. Hörmander.

Mathematics subject classification (2020): 22E45, 22E70, 81R05, 81S10, 81R30.

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