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SINGLY GENERATED SELFADJOINT–IDEAL OPERATOR SEMIGROUPS: SPECTRAL DENSITY OF THE GENERATOR AND SIMPLICITY

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Abstract. This extends our new study of the automatic selfadjoint ideal property for $B(\mathcal{H})$-operator semigroups introduced to us by Heydar Radjavi (SI semigroups for short). Our investigation here of singly generated SI semigroups led to unexpected algebraic and analytic phenomena on the simplicity of SI semigroups and on the spectral density of their generators. In particular: the SI property yields for a hyponormal operator, zero planar area measure of its approximate point spectrum; the same for the essential spectrum of an essentially normal operator; and that SI semigroups generated by unilateral weighted shifts with periodic nonzero weights are simple. We also characterized the simplicity of the SI semigroups generated by certain commuting classes of normal operators.

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