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NONLINEAR SKEW LIE TYPE HIGHER DERIVATIONS ON SOME OPERATOR ALGEBRAS

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Abstract. Let $\mathscr{A}$ be a unital $*$-algebra. Let $p_n(A_1,A_2,\cdots,A_n)$ be the polynomial defined by n indeterminates $A_1,A_2,\cdots,A_n \in \mathscr{A}$ and their multiple skew Lie product, and $\mathbb{N}$ be the set of non-negative integers. In this paper, under some mild conditions on $\mathscr{A}$, it is shown that if $\mathscr{D} = \{d_m\}_{m\in\mathbb{N}}$ is the family of maps $d_m : \mathscr{A} \to \mathscr{A}$ such that $d_0 = id_{\mathscr{A}}$, the identity map on $\mathscr{A}$ satisfying

$$d_m(p_n(A_1,A_2,\cdots,A_n)) = \sum_{i_1+i_2+\cdots+i_n=m} p_n(d_{i_1}(A_1),d_{i_2}(A_2),\cdots,d_{i_n}(A_n))$$

for all $A_1,A_2,\cdots,A_n \in \mathscr{A}$ and for each $m \in \mathbb{N}$, then $\mathscr{D} = \{d_m\}_{m\in\mathbb{N}}$ is an additive $*$-higher derivation. Moreover, we apply the above result to prime $*$-algebras, von Neumann algebras with no central summands of type I_1, factor von Neumann algebras and standard operator algebras.

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