

## THE COMPLETE NEVANLINNA–PICK PROPERTY FOR BEURLING TYPE QUOTIENT MODULES OVER THE BIDISK

JIMING SHEN AND YIXIN YANG\*

**Abstract.** In this paper, we show that for the inner function  $\theta(z_1, z_2) \in H^2(\mathbb{D}^2)$ , the Beurling type quotient module  $\mathcal{X}_\theta = H^2(\mathbb{D}^2) \ominus \theta H^2(\mathbb{D}^2)$  over the bidisk has complete Nevanlinna–Pick property if and only if  $\theta$  is a one-variable Möbius map.

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