

## BOUNDS FOR THE EIGENVALUES OF NON-MONIC OPERATOR POLYNOMIALS

MUQILE GAO, DEYU WU\* AND ALATANCANG CHEN

*Abstract.* In this paper, we give some new bounds for the eigenvalues of non-monic operator polynomials by applying several numerical radius inequalities to the Frobenius companion matrices of these polynomials. Our bounds refine the existing bounds for monic matrix polynomials.

*Mathematics subject classification (2020):* 47A12, 30C15.

*Keywords and phrases:* Operator polynomial, numerical radius, companion matrix, eigenvalue.

### REFERENCES

- [1] A. ABU-OMAR AND F. KITTANEH, *Estimates for the numerical radius and the spectral radius of the Frobenius companion matrix and bounds for the zeros of polynomials*, Annals of Functional Analysis. **5**, (2014), 56–62.
- [2] A. AHMAD, O. HIRZALLAH AND F. KITTANEH, *Singular value and norm inequalities for products and sums of matrices*, Periodica Mathematica Hungarica. **88**, (2024), 204–217.
- [3] W. BANI-DOMI AND F. KITTANEH, *Numerical radius inequalities for operator matrices*, Linear and Multilinear algebra. **57**, (2009), 421–427.
- [4] T. BOTTAZZI AND C. CONDE, *Generalized Buzano's Inequality*, arXiv:2204.14233. (2022).
- [5] M. L. BUZANO, *Generalizzazione della diseguaglianza di Cauchy-Schwarz*, Rend. Sem. Mat. Univ. e Politech. Torino. **31** (1), (1974), 405–409.
- [6] M. GAO, D. Y. WU AND A. CHEN, *Generalized upper bounds estimation of numerical radius and norm for the sum of operators*, Mediterranean Journal of Mathematics. **20**, (2023), 210.
- [7] H. GUELFEN AND F. KITTANEH, *On numerical radius inequalities for operator matrices*, Numerical Functional Analysis and Optimization. **40**, (2019), 1231–1241.
- [8] N. J. HIGHAM AND F. TISSEUR, *Bounds for eigenvalues of matrix polynomials*, Linear algebra and its applications. **358**, (2003), 5–22.
- [9] A. JARADAT AND F. KITTANEH, *Bounds for the eigenvalues of monic matrix polynomials from numerical radius inequalities*, Advances in Operator Theory. **5**, (2020), 734–743.
- [10] F. KITTANEH, *A numerical radius inequality and an estimate for the numerical radius of the Frobenius companion matrix*, Studia Math. **158**, (2003), 11–17.
- [11] X. LIU AND D. Y. WU, *The essential spectrum equalities of  $2 \times 2$  unbounded upper triangular operator matrices*, Operators and Matrices. **17**, (2023), 1065–1076.
- [12] R. F. LI, D. Y. WU AND A. CHEN, *Some general quadratic numerical radius inequalities for the off-diagonal parts of  $2 \times 2$  block operator matrices*, Operators and Matrices. **15**, (2021), 1427–1444.
- [13] A. MELMAN, *Polynomial eigenvalue bounds from companion matrix polynomials*, Linear and Multilinear Algebra. **67**, (2019), 598–612.
- [14] M. ODEH, *Bounds for the zeros of polynomials based on certain matrix inequalities*, Advances in Mathematics: Scientific Journal. **10**, (2021), 819–839.
- [15] K. PAUL AND S. BAG, *Estimation of bounds for the zeros of a polynomial using numerical radius*, Applied Mathematics and Computation. **222**, (2013), 231–243.