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OPPENHEIM–SCHUR'S INEQUALITY AND RKHS

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Abstract. In 2012, we obtained Oppenheim's inequality for positive semidefinite matrices and its equality condition by the reproducing kernel method. In this paper, as a continuation, we give a reproducing kernel proof of the block matrix version of the Oppenheim-Schur's inequality and its equality condition in the positive definite case.

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