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WEIGHTED SUBSEQUENTIAL ERGODIC THEOREMS ON ORLICZ SPACES

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Abstract. For a semifinite von Neumann algebra M, individual convergence of subsequential, $\mathscr{Z}(M)$ (center of M) valued weighted ergodic averages is studied in non commutative Orlicz spaces. In the process, we also derive a maximal ergodic inequality corresponding to such averages in noncommutative L^p $(1 \leqslant p < \infty)$ spaces using the weak $(1,1)$ inequality obtained by Yeadon.

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