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MAXIMAL DIMENSION OF AFFINE SUBSPACES OF SPECIFIC MATRICES

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Abstract. For every $n \in \mathbb{N}$ and every field K, let $M(n \times n, K)$ be the set of $n \times n$ matrices over K, let $N(n,K)$ be the set of nilpotent $n \times n$ matrices over K and let $D(n,K)$ be the set of $n \times n$ matrices over K which are diagonalizable over K, that is, which are diagonalizable in $M(n \times n, K)$. Moreover, if K is a field with an involutory automorphism, let $R(n,K)$ be the set of normal $n \times n$ matrices over K.

In this short note we prove that the maximal dimension of an affine subspace in $N(n,K)$ is $\frac{n(n-1)}{2}$ and, if the characteristic of the field is zero, an affine not linear subspace in $N(n,K)$ has dimension less than or equal to $\frac{n(n-1)}{2} - 1$. Moreover we prove that the maximal dimension of an affine subspace in $R(n,\mathbb{C})$ is n, the maximal dimension of a linear subspace in $D(n,\mathbb{R})$ is $\frac{n(n+1)}{2}$, while the maximal dimension of an affine not linear subspace in $D(n,\mathbb{R})$ is $\frac{n(n+1)}{2} - 1$.

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REFERENCES

[1] H. FLANDERS, *On spaces of linear transformations with bounded rank*, J. London Math. Soc., **37** (1962), pp. 10–16.

[2] F. R. GANTMACHER, *The Theory of Matrices*, vol. 2 Chelsea, Publishing Company New York 1959.

[3] G. HARRIS, C. MARTIN, *The roots of a complex polynomial vary continously as a function of the coefficients*, Proc. A.M.S., **100** (1987), pp. 390–392.

[4] W. V. D. HODGE, D. PEDOE, *Methods of Algebraic Geometry*, vol. 2, Cambridge University Press-Cambidge 1994.

[5] B. ILIC, J. M. LANDSBERG, *On symmetric degeneracy loci, spaces of symmetric matrices of constant rank and dual varieties*, Mathematischen Annlen, **314** (1999), pp. 159–174.

[6] C. DE SEGUINS PAZZIS, *Large affine spaces of non-singular matrices*, Trans. A.M.S., **365** (2013), pp. 2569–2596.

[7] C. DE SEGUINS PAZZIS, *On the matrices of given rank in a large subspace*, Linear Algebra Appl., **435** (2011), pp. 147–151.

[8] C. DE SEGUINS PAZZIS, *On affine spaces of alternating matrices with constant rank*, Linear and Multilinear Algebra, published on line, (2024), https://doi.org/10.1080/03081087.2024.2356827.

[9] C. DE SEGUINS PAZZIS, *On affine spaces of rectangular matrices with constant rank*, arXiv:2405.02689.

[10] M. GERSTENHABER, *On Nilalgebras and Linear Varieties of Nilpotent Matrices, I*, Amer. J. Math., **80** (1958), pp. 614–622.

[11] B. MATHES, M. OMLADIČ, H. RADJAVI, *Linear subspaces of nilpotent matrices*, Linear Algebra Appl., **149** (1991), pp. 215–225.

[12] R. QUINLAN, *Spaces of matrices without non-zero eigenvalues in their field of definition, and a question of Szechtman*, Linear Algebra Appl., **434** (2011), pp. 1580–1587.

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[13] E. RUBEI, *Affine subspaces of matrices with constant rank*, Linear Algebra Appl., **644** (2022), pp. 259–269.

[14] E. RUBEI, *Affine subspaces of antisymmetric matrices with constant rank*, Linear and Multilinear Algebra, **72** (2024), pp. 1741–1750.

[15] E. RUBEI, *Affine subspaces of matrices with rank in a range*, arXiv:2405.04694, to appear in The Electronic Journal of Linear Algebra.

[16] V. N. SEREZHKIN, *Linear transformations preserving nilpotency* (Russian), Vestsi Akad. Navuk BSSR Ser. Fīz.-Mat. Navuk, **6** (1985), pp. 46–50, 125.

[17] R. WESTWICK, *Spaces of matrices of fixed rank*, Linear and Multilinear Algebra, **20** (1987), pp. 171–174.