Operators and Matrices
Volume 18, Number 4 (2024), 789–803 doi:10.7153/oam-2024-18-48

SEVERAL PROPERTIES OF THE SPECTRUM AND LOCAL SPECTRUM OF CLASS A_n OPERATORS

M. H. M. RASHID AND ATSUSHI UCHIYAMA

Abstract. In this article, we establish some conditions which imply the normality of class A_n. Also, we prove that if T is a class A_n and $\mathcal{M}$ is an invariant subspace of T such that $T|_{\mathcal{M}}$ is a normal operator with $0 \notin \sigma_p(T|_{\mathcal{M}})$, then $\mathcal{M}$ reduces T. Moreover, we show that Weyl's theorem holds for every class A_n operator and some results related to the Riesz idempotent of class A_n operators. By using the spectral properties of class A_n operators, we prove that a class A_n contraction is the direct sum of a unitary and a $C_{\cdot 0}$ completely non-unitary contraction. In addition, the existence of a nontrivial hyperinvariant subspace of a class A_n operator will be shown.

Mathematics subject classification (2020): 47A10, 47A12, 47B20.
Keywords and phrases: Class A_n operators, Weyl's theorem, Riesz idempotent.

REFERENCES

[1] P. AIENA, *Fredholm and local spectral theory with applications to multipliers*, Kluwer, 2004.
[2] P. AIENA, E. APONTE AND E. BALZAN, *Weyl type theorems for left and right polaroid operators*, Integral Equations Operator Theory **66** (2010), 1–20.
[3] A. BACHIR AND P. PAGACZ, *An asymmetric Putnam-Fuglede theorem for $*$-paranormal operators*, arXiv:1405.4844 [math.FA].
[4] S. K. BERBERIAN, *Approximate proper vectors*, Proc. Amer. Math. Soc. **13** (1962), 111–114.
[5] C. A. MC CARTHY, C_p, Iseral J. Math. **5** (1967), 249–271.
[6] L. A. COBURN, *Weyl's theorem for nonnormal operators*, Michigan Math. J. **13** (1966), 285–288.
[7] M. CHŌ AND S. ÔTA, *On n-paranormal operators*, J. Math. Research **5** (2013), no. 2, 107–114.
[8] J. B. CONWAY, *Subnormal Operators*, Pitnam, Boston, 1981.
[9] E. DURSZT, *Contractions as restricted shifts*, Acta Scientiarum Mathematicarum **48** (1–4) (1985), 129–134.
[10] M. FUJII, S. IZUMINO, AND R. NAKAMOTO, *Classes of operators determined by the Heinz-Kato-Furuta inequality and the Hölder-McCarthy inequality*, Nihonkai Math. J. **5** (1994), 61–67.
[11] T. FURUTA, *On the Class of Paranormal operators*, Proc. Jaban. Acad. **43** (1967), 594–598.
[12] T. FURUTA, *$A \geqslant B \geqslant O$ assures $(B^rA^pB^r)^{\frac{1}{q}} \geqslant B^{\frac{p+2r}{q}}$ for $r \geqslant 0$, $p \geqslant 0$, $q \geqslant 1$ with $(1+2r)q \geqslant (p+2r)$*, Proc. Amer. Math. Soc. **101** (1987), 85–88.
[13] T. FURUTA, M. ITO AND T. YAMAZAKI, *A subclass of paranormal operators including class of log-hyponormal and several related classes*, Sci. math. **1** (1998), 389–403.
[14] P. R. HALMOS, *A Hilbert Space Problem Book*, Van Nostrand, Princeton, 1967.
[15] P. R. HALMOS, *A Hilbert space problem Book*, second edition, New York, Springer-Verlag 1982.
[16] F. HANSEN, *An equality*, Math. Ann. **246** (1980), 249–250.
[17] I. H. JEON AND I. H. KIM, *On operators satisfying $T^*|T^2|T \geqslant T^*|T|^2T^*$*, Linear Alg. Appl. **418** (2006), 854–862.
[18] I. B. JUNG, E. KO, C. PEARCY, *Aluthge transforms of operators*, Integral Equation Operator Theory **37** (2000), 437–448.
[19] C. S. KUBRUSLY, P. C. M. VIEIRA, AND D. O. PINTO, *A decomposition for a class of contractions*, Advances in Mathematical Sciences and Applications **6** (2) (1996), 523–530.

Paper OaM-18-48

[20] K. B. Laursen and M. M. Neumann, *An Introduction to Local Spectral Theory*, The Clarendon Press, Oxford University Press, New York, 2000.

[21] R. Lange and S. Wang, *New Approaches in Spectral Decomposition*, Contemp. Math. **128**, Amer. Math. Society, 1992.

[22] S. Mecheri, *Fuglede-Putnam theorem for class A operators*, Colloquium Math. **138** (2) (2015), 183–191.

[23] S. Panayappan, N. Jayanthi and D. Sumathi, *Weyl's theorem and Tensor product for class A_k operators*, Pure Math. Sci. **1** (1) (2012), 13–23.

[24] M. Radjabalipour, *On majorization and normality of operators*, Proc. Amer. Math. Soc. **62** (1977), 105–110.

[25] M. H. M. Rashid, *On k-quasi-$*$-paranormal operators*, RACSAM Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. **110** (2) (2016), 655–666.

[26] M. H. M. Rashid, *Some invariant subspaces for w-hyponormal operators*, Linear and Multilinear Algebra **67**: 7 (2019), 1460–1470, doi:10.1080/03081087.2018.1455803.

[27] M. H. M. Rashid, *Spectrum of k-quasi-class A_n operators*, New Zealand Journal of Mathematics. **50** (2020), 61–70.

[28] M. Schechter, *Principles of functional analysis*, Acadmic Press Inc., New York, 1971.

[29] J. G. Stampfli, *Hyponormal operators and spectrum density*, Trans. Amer. Math. Soc. **117** (1965), 469–476.

[30] A. Uchiyama and K. Tanahashi, *A note on $*$-paranormal operators and related classes of operators*, Bull. Korean Math. Soc. **51** (2014), 357–371.

[31] A. Uchiyama, *On isolated points of the spectrum of paranomal operators*, Integral Equations Operator Theory **55** (2006), 145–151.

[32] Mi Young Lee and Sang Hun Lee, *On a class of operators related to paranormal operators*, J. Korean Math. Soc. **44** (1) (2007), 25–34.

[33] J. Yuan and Z. Gao, *Weyl spectrum of class A_n and n-paranormal operators*, Integral Equations Operator Theory **60** (2008), 289–298.

[34] J. T. Yuan, G. X. Ji, *On (n,k)-quasiparanormal operators*, Studia Math. **209** (2012), 289–301, doi:10.4064/sm209-3-6.