Operators and Matrices
Volume 18, Number 4 (2024), 833–843
doi:10.7153/oam-2024-18-50

REDUCING SUBSPACES OF SKEW SYMMETRIC OPERATORS

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Abstract. An operator T on a complex, separable Hilbert space $\mathscr{H}$ is said to be skew symmetric if there exists a conjugation C on $\mathscr{H}$ such that $CTC = -T^*$. This paper aims to describe reducing subspaces of skew symmetric operators from the view point of approximation. In particular, given a skew symmetric operator T, $1 \leqslant n \leqslant \aleph_0$ and $\varepsilon > 0$, it is proved that there exists a compact operator K with $\|K\| < \varepsilon$ such that $T + K$ is skew symmetric and has exactly n minimal reducing subspaces.

Mathematics subject classification (2020): Primary 47B99, 47A65; Secondary 47A58.
Keywords and phrases: Skew symmetric operators, reducing subspaces, compact perturbation, irreducible operators.

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