Operators and Matrices
Volume 18, Number 4 (2024), 845–853
doi:10.7153/oam-2024-18-51

EIGENVALUES OF INDEFINITE q–STURM–LIOUVILLE PROBLEM WITH q–COUPLED BOUNDARY CONDITION

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Abstract. The present paper deals with non-real eigenvalues of indefinite q-Sturm-Liouville problems with coupled boundary condition

$$\begin{cases} -\frac{1}{q}D_{q^{-1}}D_q y(x) + v(x)y(x) = \lambda w(x)y(x), \\ \begin{pmatrix} y(1) \\ D_{q^{-1}}y(1) \end{pmatrix} = K \begin{pmatrix} y(0) \\ D_{q^{-1}}y(0) \end{pmatrix}. \end{cases}$$

The upper bounds on the imaginary and real parts of non-real eigenvalues for this indefinite q-Sturm-Liouville problem are obtained in terms of the coefficients v, w and the q-coupled boundary conditions. This is a challenging open problem according to the regular indefinite Sturm-Liouville problems and there has been little research on this problems so far. A priori bounds on the non-real eigenvalues in this paper can of course be combined with other estimates of the indefinite q-Sturm-Liouville problems under the assumption in this paper and the methods partly inspired by the estimates for nonlocal regular indefinite Sturm-Liouville problems with nonlocal coupled boundary conditions.

Mathematics subject classification (2020): 39A13, 34B05, 34B24, 34L15, 47B50.
Keywords and phrases: Indefinite q-Sturm-Liouville problem, non-real eigenvalues, upper bounds, q-coupled boundary condition.

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