MULTIPLE–TERM REFINEMENTS FOR UPPER BOUND OF THE HERMITE–HADAMARD INEQUALITY AND APPLICATIONS

TRAN DINH PHUNG, NGUYEN NGOC HUE* AND DUONG QUOC HUY

Abstract. In this work, we propose various new upper bounds of the Hermite-Hadamard inequality for convex and radical convex functions via the linear and quadratic interpolations of convex functions. Some applications of the newly introduced results to operator inequalities, numerical radius and unitarily invariant norm inequalities are also given.

Mathematics subject classification (2020): 15A39, 26A51, 47A30, 47A63.

Keywords and phrases: Hermite-Hadamard inequality, convex function, quadratic interpolation, unitarily invariant norm, operator inequalities.

REFERENCES

[1] R. BHATIA, *Matrix Analysis*, Springer, New York, 1997.

[2] R. BHATIA, C. DAVIS, *More matrix forms of the arithmetic-geometric mean inequality*, SIAM J. Matrix Anal. Appl., **14**, (1993), 132–136.

[3] P. BHUNIA, K. PAUL, *Refinements of norm and numerical radius inequalities*, Rocky Mountain J. Math., **51**, 6 (2021), 1953–1965.

[4] D. CHOI, M. KRNIĆ, J. PEČARIĆ, *Improved Jensen-type inequalities via linear interpolation and applications*, J. Math. Inequal., **11**, (2017), 301–322.

[5] S. S. DRAGOMIR, *Bounds for the normalised Jensen functional*, Bull. Austral. Math. Soc., **74**, 3 (2006), 471–478.

[6] S. S. DRAGOMIR, *Hermite-Hadamard type inequalities for operator convex functions*, Appl. Math. Comput., **218**, (2011), 766–772.

[7] J. HADAMARD, *Étude sur les propriétés des fonctions entiéres et en particulier d'une fonction considérée par Riemann'*, J. Math. Pures Appl., **58**, (1893), 171–215.

[8] P. C, HAMMER, *The midpoint method of numerical integration*, Math. Mag., **31**, (1957/1958), 193–195.

[9] CH. HERMITE, *Sur deux limites d'une intégrale dé finie*, Mathesis, **3**, 1883, 82.

[10] D. Q. HUY, T. T. QUANG, D. T. T. VAN, *Improved Jensen-type inequalities via quadratic interpolation and applications*, J. Math. Inequal., **16**, (2022), 811–826.

[11] T. KATO, *Notes on some inequalities for linear operators*, Math. Ann., **125**, (1952), 208–212.

[12] F. KITTANEH, *On the convexity of the Heinz means*, Integr. Equ. Oper. Theory, **68**, (2010), 519–527.

[13] P. R. MERCER, *Hadamard's inequality and trapezoid rules for the Riemann-Stieltjes integral*, J. Math. Anal. Appl., **344**, (2008), 921–926.

[14] M. MERKLE, Z. D. MITROVIĆ, *A tight Hermite-Hadamard inequality and a generic method for comparison between residuals of inequalities with convex functions*, Period Math. Hung., **85**, 1 (2022), 32–44.

[15] F. MITROI, *About the precision in Jensen-Steffensen inequality*, Ann. Univ. Craiova Math. Comput. Sci. Ser., **37**, 4 (2010), 73–84.

[16] H. R. MORADI, M. SABABHEH, *More accurate numerical radius inequalities (II)*, Linear Algebra Appl., **69**, 5 (2021), 921–933.

[17] C. P. NICULESCU, L.-E. PERSSON, *Old and new on the Hermite-Hadamard inequality*, Real Anal. Exch., **29**, 2 (2004), 663–686.

[18] A. OLBRYŚ, T. SZOSTOK, *Inequalities of the Hermite-Hadamard type involving numerical differentiation formulas*, Results Math., **67**, (2015), 403–416.

[19] M. E. OMIDVAR, H. R. MORADI, *Better bounds on the numerical radii of Hilbert space operators*, Linear Algebra Appl., **604**, (2020), 265–277.

[20] M. SABABHEH, *Numerical radius inequalities via convexity*, Linear Algebra Appl., **549**, (2018), 67–78.

[21] M. SABABHEH, S. FURUICHI, Z. HEYDARBEYGI, H. R. MORADI, *On the arithmetic-geometric mean inequality*, J. Math. Inequal., **15**, 3 (2021), 1255–1266.

[22] M. SABABHEH, S. FURUICHI, H. R. MORADI, *Improved matrix inequalities using radical convexity*, J. Math. Inequal., **17**, 1 (2023), 249–236.

[23] M. SABABHEH, H. R. MORADI, *More accurate numerical radius inequalities (I)*, Linear Multilinear Algebra, **69**, 10 (2021), 1964–1973.

[24] M. SABABHEH, H. R. MORADI, *Radical convex functions*, Mediterr. J. Math., **18**, 4 (2021), 137.

[25] S. TAFAZOLI, H. R. MORADI, S. FURUICHI, P. HARIKRISHNAN, *Further inequalities for the numerical radius of Hilbert space operators*, J. Math. Inequal., **13**, 4 (2019), 955–967.

[26] S. WASOWICZ, A. WITKOWSKI, *On some inequality of Hermite-Hadamard type*, Opuscula Math., **32**, 3 (2012), 591–600.

[27] G. ZABANDAN, *A new refinement of the Hermite-Hadamard inequality for convex functions*, J. Inequal. Pure Appl. Math., **10**, 2 (2009) 7.