

FURTHER INEQUALITIES FOR NORMAL MATRICES

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Abstract. In this paper, we study the properties of normal matrices and obtain some nonnegative function inequalities between normal matrices and their modulus. Some related works are also presented. Furthermore, we also give a novel approach to prove

$$\left\| f\left(\frac{1}{2}\begin{pmatrix} |A|+|B| & A^*+B^* \\ A+B & |A^*|+|B^*| \end{pmatrix}\right)\right\| \leq \|f(|A|) + f(|B|)\|$$

for normal matrices A, B .

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