ON APPROXIMATE AND ACTUAL REDUCIBILITY OF MATRIX GROUPS

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Abstract. We introduce the notions of ε-approximate fixed point and weak ε-approximate fixed point. We show that for a group of unitary matrices even the existence of a nontrivial weak ε-approximate fixed point for sufficiently small ε gives an actual nontrivial common eigenvector. We give estimates for ε in terms of the size n of matrices and prove that the dependence is polynomial. Moreover, we show that the common eigenvector is polynomially close to the starting weak approximate fixed point.

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REFERENCES

[1] B. BEKKA, P. DE LA HARPE, A. VALETTE, *Kazhdan's Property (T)*, Cambridge University Press (2008).

[2] J. BERNIK, R. GURALNICK, M. MASTNAK, *Reduction theorems for groups of matrices*, Linear Algebra Appl., **383** (2004), 119–126.

[3] J. BERNIK, H. RADJAVI, *How small can nonzero commutators be?*, Indiana University Math. J., **54**, (2005), no. 2, 309–320.

[4] N. BOURBAKI, *Groupes et algèbres de Lie*, Chapitre 9, Groupes de Lie réels compacts, Springer-Verlag, Berlin (2007).

[5] The GAP Group, *GAP – Groups, Algorithms, and Programming*, Version 4.10.1; 2019, https://www.gap-system.org.

[6] G. H. HARDY, J. E. LITTLEWOOD, AND G. PÓLYA, *Some simple inequalities satisfied by convex functions*, Messenger of Math., **58** (1929), 145–152.

[7] G. H. HARDY, J. E. LITTLEWOOD, AND G. PÓLYA, *Inequalities*, Cambridge University Press, Cambridge (1934).

[8] D. KAZHDAN, *Connection of the dual space of a group with the structure of its closed subgroups*, Funct. Anal. Appl., **1**, (1967), 63–65.

[9] B. KUZMA, M. MASTNAK, M. OMLADIČ, AND H. RADJAVI, *On approximate versions of reducibility results for matrix groups and semigroups*, Linear Algebra Appl., **577**, (2019), 41–52.

[10] M. W. LIEBECK, E. A. O'BRIEN, A. SHALEV, AND P. H. TIEP, *The Ore conjecture*, J. Eur. Math. Soc., **12**, (2010), 939–1008.

[11] M. MASTNAK, H. RADJAVI, *Matrix semigroups whose ring commutators have real spectra are realizable*, Semigroup Forum, **95**, (2017), 51–65.

[12] H. RADJAVI AND P. ROSENTHAL, *Simultaneous Triangularization*, Springer-Verlag, New York, (2000).

[13] J.-P. SERRE, *Représentations linéaires des groupes finis*, Hermann, (1998).

[14] D. SUPRUNENKO, *Soluble and Nilpotent Linear Groups*, Translations of Mathematical Monographs, vol. 9, American Mathematical Society, Providence, Rhode Island, (1963).

[15] P. TAUVEL AND R. W. T. YU, *Lie algebras and Algebraic groups*, Springer Monographs in Mathematics, Springer, New York, (2005).