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MAPS PRESERVING THE LOCAL SPECTRAL SUBSPACE OF SKEW–PRODUCT OF OPERATORS

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Abstract. Let $B(H)$ be the algebra of all bounded linear operators on an infinite-dimensional complex Hilbert space H. For $T \in B(H)$ and $\lambda \in \mathbb{C}$, let $H_T(\{\lambda\})$ denotes the local spectral subspace of T associated with $\{\lambda\}$. We prove that if $\varphi : B(H) \to B(H)$ be an additive map such that its range contains all operators of rank at most two and satisfies

$$H_{\varphi(T)\varphi(S)^*}(\{\lambda\}) = H_{TS^*}(\{\lambda\})$$

for all $T,S \in B(H)$ and $\lambda \in \mathbb{C}$, then there exist a unitary operator V in $B(H)$ and a nonzero scalar μ such that $\varphi(T) = \mu TV^*$ for all $T \in B(H)$. We also show if φ_1 and φ_2 be additive maps from $B(H)$ into $B(H)$ such that their ranges contain all operators of rank at most two and satisfies

$$H_{\varphi_1(T)^*\varphi_2(S)}(\{\lambda\}) = H_{T^*S}(\{\lambda\})$$

for all $T,S \in B(H)$ and $\lambda \in \mathbb{C}$. Then $\varphi_2(I)^*$ is invertible, and $\varphi_1(T) = (\varphi_2(I)^*)^{-1}T$ and $\varphi_2(T) = \varphi_2(I)T$ for all $T \in B(H)$.

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