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UNCERTAINTY PRINCIPLES FOR THE WHITTAKER WIGNER TRANSFORM

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Abstract. The Whittaker Wigner transform (WWT) is a novel addition to the class of Wigner transforms, which has gained a respectable status in the realm of time-frequency signal analysis within a short span of time. Knowing the fact that the study of the time-frequency analysis is both theoretically interesting and practically useful, the aim of this paper is to explore a class of quantitative uncertainty principles associated with the WWT, including the Heisenberg's uncertainty, Benedick's UP, Donoho-Stark's UP, Benedick-Amrein-Berthier UP and the uncertainty principle for orthonormal sequences.

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