Operators and Matrices
Volume 19, Number 1 (2025), 13–29
doi:10.7153/oam-2025-19-02

NOTE ON BOUNDS FOR SECOND EXTREME EIGENVALUES OF HERMITIAN MATRICES

R. SHARMA, M. PAL* AND V. SHARMA

Abstract. We obtain some bounds for the second smallest and second largest eigenvalues of a Hermitian matrix. Some additional bounds for the second extreme eigenvalues of positive definite matrices are also discussed here.

Mathematics subject classification (2020): 15A42, 15B57.
Keywords and phrases: Eigenvalues, principal submatrices, positive linear functionals, spread, non-negative symmetric matrix.

REFERENCES

[1] M. ANDELIĆ, T. KOLEDIN AND Z. STANIĆ, *Nested graphs with bounded second (signless Laplacian) eigenvalue*, Electron. J. Linear Algebra, **24** (2012): 181–201.
[2] M. ANDELIĆ, S. K. SIMIĆ, AND D. ZIVKOVIĆ, *Reflexive line graphs of trees and Salem numbers*, Mediterranean J. Math., **16** (2019): 1–16.
[3] A. BERMAN, M. FARBER, *A lower bound for the second largest Laplacian eigenvalue of weighted graphs*, Electron. J. Linear Algebra, **22** (2011): 1179–1184.
[4] R. BHATIA, *Matrix Analysis*, Springer, New York, (1997).
[5] R. BHATIA, *Positive Definite Matrices*, Princeton University Press, (2007).
[6] R. BHATIA AND C. DAVIS, *A better bound on the variance*, Amer. Math. Monthly, **107** (2000): 353–357.
[7] R. BHATIA AND R. SHARMA, *Positive linear maps and spreads of matrices*, Amer. Math. Monthly, **121** (2014): 619-624.
[8] R. BHATIA AND R. SHARMA, *Positive linear maps and spreads of matrices II*, Linear Algebra Appl., **491** (2016): 30–40.
[9] R. BHATIA AND R. SHARMA, *Eigenvalues and diagonal elements*, Indian J. Pure and Appl. Math., **54** (3) (2023): 757–759.
[10] Z. B. CHARLES, M. FARBER, C. R. JOHNSON AND L. K. SHAFFER, *The relation between the diagonal entries and eigenvalues of a symmetric matrix, based upon the sign patterns of its off-diagonal entries*, Linear Algebra Appl., **438** (2013): 1427–1445.
[11] S. DRURY, *Graphs with second signless Laplacian eigenvalue* $\leqslant 4$, Spec. Matrices, **10** (1) (2022): 131–152.
[12] M. FIEDLER, *Algebraic connectivity of graphs*, Czechoslovak Math. J., **23** (2) (1973): 298–305.
[13] E. HOPF, *An inequality for positive linear integral operators*, J. Math. Mech., **12** (5) (1963): 683–692.
[14] R. A. HORN AND C. R. JOHNSON, *Matrix Analysis*, Cambridge University Press, (2013).
[15] E. JIANG AND X. ZHAN, *Lower bounds for the spread of a Hermitian matrix*, Linear Algebra Appl., **256** (1997): 153–163.
[16] R. V. KADISON, *A generalized Schwarz inequality and algebraic invariants for operator algebras*, Ann. of Math., **56** (3) (1952): 494–503.
[17] T. KOLEDIN AND Z. STANIĆ, *Regular graphs with small second largest eigenvalue*, Appl. Anal. Discrete Math., **7** (2013): 235–249.
[18] R. KUMAR AND R. SHARMA, *Some inequalities involving eigenvalues and positive linear maps*, Adv. Oper. Theory, **8** (3) (2023): 42.

Paper OaM-19-02

[19] M. LIU, C. CHEN, AND Z. STANIĆ, *Connected (K4-e)-free graphs whose second largest eigenvalue does not exceed 1*, European J. Combin., **115** (2024): 103775.
[20] J. NAGY, *Über algebraische Gleichungen mit lauter reellen Wurzeln*, Jahresber. Deutsch. Math.-Verein., **27** (1918): 37–43.
[21] A. NEUMAIER, *The second largest eigenvalue of a tree*, Linear Algebra Appl., **46** (1982): 9–25.
[22] S. K. SIMIĆ, M. ANDELIĆ, C. M. DA FONSECA, D. ZIVKOVIĆ, *Notes on the second largest eigenvalue of a graph*, Linear Algebra Appl., **465** (2015): 262–274.
[23] S. K. SIMIĆ, D. ZIVKOVIĆ, M. ANDELIĆ AND C. M. DA FONSECA, *Reflexive line graphs of trees*, J. Algebraic Combin., **43** (2016): 447–464.
[24] R. SHARMA AND M. PAL, *Note on bounds for eigenvalues using traces*, Oper. Matrices, **16** (3) (2022): 759–773.
[25] R. SHARMA, M. PAL AND A. SHARMA, *Determinant and eigenvalue inequalities involving nonnegative matrices*, Adv. Oper. Theory, **8** (3) (2023): 55.
[26] Z. STANIĆ, *Lower bounds for the algebraic connectivity of graphs with specified subgraphs*, Electron. J. Graph Theory Appl., **9** (2) (2021): 257–264.
[27] H. WOLKOWICZ AND G. P. H. STYAN, *Bounds for eigenvalues using traces*, Linear Algebra Appl., **29** (1980): 471–506.
[28] C. W. WU, *Algebraic connectivity of directed graphs*, Linear Multilinear Algebra, **53** (2005): 203–223.
[29] Z. XIAODONG AND L. JIONGSHENG, *On the k-th largest eigenvalue of the Laplacian matrix of a graph*, Acta Math. Appl. Sin., **17** (2) (2001): 183–190.