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# SUFFICIENT CONDITIONS FOR FACTOR POSETS OF FRAMES IN $\mathbb{R}^n$ AND THEIR GRAPH ASSOCIATIONS

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*Abstract.* A *frame* in $\mathbb{R}^n$ is a possibly redundant set of vectors $\{f_i\}_{i\in I}$ that span $\mathbb{R}^n$. A *tight frame* in $\mathbb{R}^n$ is a generalization of an orthonormal basis. A *factor poset* $P$ of a frame is the collection of subsets of $I$, ordered by inclusion, such that $J \subseteq I$ is in $P$ if and only if $\{f_j\}_{j\in J}$ is a tight frame. In [8], the authors studied the conditions for a given poset of index sets to be the factor poset of a frame. They gave a complete characterization of this "inverse factor poset problem" for $\mathbb{R}^2$ and a necessary condition for solving this problem in $\mathbb{R}^n$. In this paper we give sufficient conditions on poset $P \subseteq 2^I$ to be a factor poset of a frame and discuss some combinatorial conditions that are necessary for $\mathbb{R}^n$. We also study how to associate tight frames to the vertices of a given graph $G$ such that $G$ becomes the *intersection graph* of the resulting frame. By establishing a connection between poset characteristics and graph theory, we generate new tight frames. Further we establish the connection between the independence number of a graph and the maximum number of mutually disjoint index sets of prime tight subframes. We also provide an estimation of the size of the factor poset of a frame when the corresponding graph is a complete t-partite graph.

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