

## COMPACT WEIGHTED COMPOSITION-DIFFERENTIATION OPERATORS OF ORDER $n$ ON THE HARDY SPACE

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**Abstract.** The weighted composition–differentiation operator of order  $n$  is denoted by  $D_{\psi, \varphi, n}$ . In this paper, we investigate some basic properties of compact weighted composition–differentiation operators of order  $n$  on the Hardy space. Moreover, we obtain the upper estimate on the norm of the operator  $D_{\psi, \varphi, n}$ , in the case that  $\|\varphi\|_{\infty} < 1$ .

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