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PROPERTIES OF INTEGRAL OPERATORS ON BERGMAN–MORREY SPACES

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Abstract. For $0 < q < \infty$ and $0 < \eta < \infty$, the tent space $T_{q,\eta}(\mu)$ consists of all μ-measurable functions f such that

$$\|f\|^q_{T_{q,\eta}(\mu)} := \sup_{I \subseteq \partial\mathbb{D}} \frac{1}{|I|^\eta} \int_{S(I)} |f(z)|^q d\mu(z) < \infty.$$

In this note, we study the boundedness and compactness of the inclusion mapping i from Bergman-Morrey Spaces $\mathscr{A}^{p,\lambda}$ to Tent Spaces $T_{q,\eta}(\mu)$. The boundedness and essential norm of Volterra integral operators from Bergman-Morrey Spaces $\mathscr{A}^{p,\lambda}$ to Bergman-Morrey Spaces $\mathscr{A}^{q,\eta}$ are also investigated in this paper, which generalized the main results in [31]. In the end, we investigated the closed range Volterra integral operators on Bergman-Morrey Spaces $\mathscr{A}^{p,\lambda}$.

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