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ON K–FRAMES GENERATED BY OPERATORS ON HILBERT SPACES

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Abstract. The aim of this paper is to analyze K-frames generated by a bounded linear operator on a separable Hilbert space $\mathscr{H}$. First, we establish some lower bounds for the norm of an operator T when the sequence $\{T^n g\}_{g\in G, n\geqslant 0}$ satisfies the lower K-frame bound for some set $G \subset \mathscr{H}$. Furthermore, we derive a necessary condition for the sequence $\{T^n g\}_{g\in G, n\geqslant 0}$ to be a K-frame. As a consequence, we prove that the hypercyclic operator T with a hypercyclic vector in the range of K cannot generate a K-frame. Additionally, under certain conditions, we construct a Parseval iterative K-frame using an operator. Finally, we determine the form of the K-dual for K-frames generated by an operator.

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