Operators and Matrices
Volume 19, Number 3 (2025), 393–410
doi:10.7153/oam-2025-19-25

THE HILBERT MATRIX OPERATOR ACTING ON SPACES OF BOUNDED ANALYTIC FUNCTIONS

YUTING GUO AND PENGCHENG TANG*

Abstract. It is well known that the Hilbert matrix operator $\mathscr{H}$ is bounded from H^∞ to the mean Lipschitz spaces $\Lambda^p_{1/p}$ for all $1 < p < \infty$. In this paper, we prove that the range of $\mathscr{H}$ acting on H^∞ is contained in a certain Zygmund-type space, denoted by $\Lambda^{1,*}_1$. We also provide explicit upper and lower bounds for the norm of $\mathscr{H}$ as an operator from H^∞ to $\Lambda^{1,*}_1$. Moreover, we characterize the positive Borel measures μ for which the generalized Hilbert matrix operator $\mathscr{H}_\mu$ is bounded from H^∞ to the Hardy space H^q.

Mathematics subject classification (2020): 47B35, 30H05, 30H10.
Keywords and phrases: Hilbert matrix operator, bounded analytic function, norm, Carleson measure.

REFERENCES

[1] I. ARÉVALO, *A characterization of the inclusions between mixed norm spaces*, J. Math. Anal. Appl. **429** (2015) 942–955.
[2] G. BAO, F. SUN, H. WULAN, *Carleson measure and the range of Cesàro-like operator acting on* H^∞, Anal. Math. Phys. **12** (2022) no. 142.
[3] G. BAO, H. WULAN, *Hankel matrices acting on Dirichlet spaces*, J. Math. Anal. Appl. **409** (2014) 228–235.
[4] C. BELLAVITA, G. STYLOGIANNIS, *Hilbert matrix operator acting between conformally invariant spaces*, Canad. Math. Bull. **68** (2025) 550–567.
[5] M. BELTRÁN-MENEU, J. BONET, E. JORDÁ, *Generalized Hilbert operators acting on weighted spaces of holomorphic functions with sup-norms*, Banach J. Math. Anal. **19** (2025) no. 10.
[6] P. BOURDON, J. SHAPIRO, AND W. SLEDD, *Fourier series, mean Lipschitz spaces, and bounded mean oscillation*, Analysis at Urbana, vol. I (Urbana, IL, 1986–1987), 81–110, London Math. Soc. Lecture Note Ser. **137**, Cambridge Univ. Press, Cambridge, 1989.
[7] V. BOŽIN, B. KARAPETROVIĆ, *Norm of the Hilbert matrix on Bergman spaces*, J. Funct. Anal. **274** (2018) 525–543.
[8] S. BUCKLEY, P. KOSKELA, D. VUKOTIĆ, *Fractional integration, differentiation, and weighted Bergman spaces*, Math. Proc. Camb. Philos. Soc. **126** (2) (1999) 369–385.
[9] C. CHATZIFOUNTAS, D. GIRELA, J. PELÁEZ, *A generalized Hilbert matrix acting on Hardy spaces*, J. Math. Anal. Appl. **413** (2014) 154–168.
[10] N. DANIKAS, A. SISKAKIS, *The Cesàro operator on bounded analytic functions*, Analysis **13** (1993) 295–299.
[11] E. DIAMANTOPOULOS, A. SISKAKIS, *Composition operators and the Hilbert matrix*, Studia Math. **140** (2000) 191–198.
[12] E. DIAMANTOPOULOS, *Hilbert matrix on Bergman spaces*, Illinois J. Math. **48** (3) (2004) 1067–1078.
[13] M. DOSTANIĆ, M. JEVTIĆ, D. VUKOTIĆ, *Norm of the Hilbert matrix on Bergman and Hardy spaces and theorem of Nehari type*, J. Funct. Anal. **254** (2008) 2800–2815.
[14] P. DUREN, B. ROMBERG, A. SHIELDS, *Linear functionals on* H^p *spaces with* $0 < p < 1$, J. Reine Angew. Math. **238** (1969) 32–60.
[15] P. DUREN, *Theory of* H^p *spaces*, Academic Press, New York, 1970.
[16] T. FLETT, *The dual of an inequality of Hardy and Littlewood and some related inequalities*, J. Math. Anal. Appl. **38** (1972) 746–765.

Paper OaM-19-25

[17] P. GALANOPOULOS, D. GIRELA, A. MAS, N. MERCHÁN, *Operators induced by radial measures acting on the Dirichlet space*, Results Math. **78** (2023) no. 106.
[18] P. GALANOPOULOS, D. GIRELA, J. PELÁEZ, A. SISKAKIS, *Generalized Hilbert operators*, Ann. Acad. Sci. Fenn. Math. **39** (1) (2014) 231–258.
[19] P. GALANOPOULOS, J. PELÁEZ, *A Hankel matrix acting on Hardy and Bergman spaces*, Studia Math. **200** (3) (2010) 201–220.
[20] D. GIRELA, N. MERCHÁN, *A generalized Hilbert operator acting on conformally invariant spaces*, Banach J. Math. Anal. Appl. **12** (2018) 374–398.
[21] D. GIRELA, N. MERCHÁN, *A Hankel matrix acting on spaces of analytic functions*, Integral Equ. Oper. Theory **89** (4) (2017) 581–594.
[22] M. JEVTIĆ, B. KARAPETROVIĆ, *Hilbert matrix on spaces of Bergman-type*, J. Math. Anal. Appl. **453** (2017) 241–254.
[23] B. ŁANUCHA, M. NOWAK, M. PAVLOVIĆ, *Hilbert matrix operator on spaces of analytic functions*, Ann. Acad. Sci. Fenn. Math. **37** (2012) 161–174.
[24] C. LIU, *Sharp Forelli-Rudin estimates and the norm of the Bergman projection*, J. Funct. Anal. **268** (2) (2015) 255–277.
[25] N. MERCHÁN, *Mean Lipschitz spaces and a generalized Hilbert operator*, Collect. Math. **70** (2019) 59–69.
[26] M. PAVLOVIĆ, *Analytic functions with decreasing coefficients and Hardy and Bloch spaces*, Proc. Edinb. Math. Soc. **56** (2013) 623–635.
[27] M. PAVLOVIĆ, *Function classes on the unit disc, An Introduction*, De Gruyter Studies in Mathematics, vol. 52, De Gruyter, Berlin, 2014.
[28] M. PAVLOVIĆ, M. MATELJEVIĆ, *L^p-behavior of power series with positive coefficients and Hardy spaces*, Proc. Amer. Math. Soc. **87** (2) (1983) 309–316.
[29] A. PERÄLÄ, *On the optimal constant for the Bergman projection onto the Bloch space*, Ann Acad. Sci. Fenn. Math. **37** (2012) 245–249.
[30] H. WIDOM, *Hankel matrices*, Trans. Amer. Math. Soc. **121** (1966) 1–35.
[31] J. XIAO, *Holomorphic Q Classes*, Springer, Lecture Notes in Math, Berlin, 2001.
[32] S. YE, Z. ZHOU, *A Derivative-Hilbert operator acting on Bergman spaces*, J. Math. Anal. Appl. **506** (2021) no. 125553.
[33] S. YE, G. FENG, *Generalized Hilbert operators acting on weighted Bergman spaces and Dirichlet spaces*, Banach J. Math. Anal. **17** (2023) no. 38.
[34] A. ZYGMUND, *Trigonometric Series*, vol. I, II. Cambridge University Press, London, 1959.