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OPERATOR RADIUS INEQUALITIES FOR SEVERAL OPERATORS ON HILBERT SPACES

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Abstract. Let $\omega_\rho(X)$ denote the ρ-operator radius of a bounded linear operator X on a finite dimensional Hilbert space $\mathscr{H}$, where $0 < \rho \leqslant 2$. In this article, we present ρ-operator radii generalizations of various numerical radius commutator inequalities, including

$$\omega(SX+XS) \leqslant 2\sqrt{2}\,\omega(S)\cdot\|X\|,$$

$$\omega(SX^*+X^*S) \leqslant 2\,\omega(S)\cdot\|X\|,$$

and the arithmetic-geometric mean inequality:

$$\omega(XSY^*) \leqslant \frac{1}{2}\,\omega\left(|X|^2S+S|Y|^2\right),$$

under various conditions on X and Y.

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