

## STRONG $L$ -LIMITED SETS OF ORDER $p$ IN THE DUAL OF BANACH LATTICES

HALIMEH ARDAKANI\* AND MARYAM S. ZABIHINPOUR JAHROMI

*Abstract.* The class of strong  $L$ -limited sets of order  $p$  in the dual of a Banach lattice is introduced and applied to provide some operator characterizations of Banach lattices with the strong limited  $p$ -Schur property and the weak  $DP^*$  property of order  $p$ . Close connections with the strong  $L$ -limited sets of order  $p$ , with the  $(V)$ -sets and with the relatively (weakly) compact sets are established. The disjoint weak  $DP^*$  property of order  $p$  and the positive strong limited  $p$ -Schur property are thoroughly considered which can be characterized by disjoint almost limited  $p$ -convergent operators.

*Mathematics subject classification (2020):* Primary 46A40; Secondary 46B40, 46B42.

*Keywords and phrases:* Weakly  $p$ -summable sequence, almost limited set, strong limited  $p$ -Schur property, almost limited  $p$ -convergent operator.

### REFERENCES

- [1] C. D. ALIPRANTIS AND O. BURKISHAW, *Positive Operators*, Academic Press, New York, London, 1978.
- [2] B. AQZZOUZ AND A. ELBOUR, *Some characterizations of almost Dunford–Pettis operators and applications*, *Positivity* **15** (2011), 369–380.
- [3] B. AQZZOUZ AND K. BOURAS, *Dunford–Pettis sets in Banach lattices*, *Acta Math. Univ. Comenianae*, **81** (2012), 185–196.
- [4] B. AQZZOUZ AND K. BOURAS,  *$L$ -sets and almost  $L$ -sets in Banach lattices*, *Quaestiones Mathematicae*, **36** (2013), 107–118.
- [5] H. ARDAKANI AND J. X. CHEN, *Positively limited sets in Banach lattices*, *J. Math. Anal. Appl.* **526** (1) (2023), 127220, <https://doi.org/10.1016/j.jmaa.2023.127220>.
- [6] H. ARDAKANI AND J. X. CHEN, *Two classes of operators related to positively limited sets on Banach lattices*, *Quaestiones Mathematicae*, **47** (3) (2024), <https://doi.org/10.2989/16073606.2023.2252180>.
- [7] H. ARDAKANI, S. M. MOSHTAGHIOUN, S. M. S. MODARRES MOSADEGH AND M. SALIMI, *The strong Gelfand–Phillips property in Banach lattices*, *Banach J. Math. Anal.* **10** (2016), 15–26.
- [8] H. ARDAKANI AND S. M. S. MODARRES MOSADEGH, *Strong relatively compact Dunford–Pettis property in Banach lattices*, *Ann. Funct. Anal.* **7** (2016), 270–280.
- [9] H. ARDAKANI AND KH. TAGHAVINEJAD, *The strong limited  $p$ -Schur property in Banach lattices*, *OaM.* **16** (2022), 811–825.
- [10] H. ARDAKANI AND F. VALI, *On almost limited  $p$ -convergent operators on Banach lattices*, *Positivity*, **28** (2024), <https://doi.org/10.1007/s11117-024-01040-9>.
- [11] G. BOTELHO AND J. L. P. LUIZ, *On the Schur, positive Schur and weak Dunford–Pettis properties in Frechet lattices*, *Ann. Fenn. Math.* **46** (2021), 633–642.
- [12] J. CASTILLO AND F. SANCHEZ, *Dunford–Pettis-like Properties of Continuous Vector Function Spaces*, *Revista Mathematica* **6** (1993), 43–59.
- [13] J. X. CHEN, Z. L. CHEN AND G. X. JI, *Almost limited sets in Banach lattices*, *J. Math. Anal. Appl.* **412** (2014), 547–563.
- [14] Z. L. CHEN AND A. W. WICKSTEAD, *Relative weak compactness of solid hulls in Banach lattices*, *Indag. Mathem. N.S.* **9** (1995), 187–196.

- [15] J. DIESTEL, *Sequences and Series in Banach Spaces*, Graduate Texts in Math. **92**, Springer-Verlag, Berlin, 1984.
- [16] P. G. DODDS AND D. H. FREMLIN, *Compact operators in Banach lattices*, Isr. J. Math. **34** (1979), 287–320.
- [17] G. EMMANUELE, *Banach spaces in which Dunford-Pettis sets are relatively compact*, Arch. Math. **58** (1992), 477–485.
- [18] G. EMMANUELE, *A dual characterization of Banach spaces not containing  $\ell_1$* , Bull. Pol. Acad. Sci. Math. **34** (1986), 155–160.
- [19] G. EMMANUELE, *On Banach spaces with the Gelfand–Phillips property, III*, J. Math. Pures Appl. **72** (1993), 327–333.
- [20] P. GALINDO AND V. C. C. MIRANDA, *Some properties of  $p$ -limited sets*, Proc. Amer. Math. Soc. **152** (2) (2024), 749–763.
- [21] P. GALINDO AND V. C. C. MIRANDA, *Grothendieck-type subsets of Banach lattices*, J. Math. Anal. Appl. **506** (2022), 1–14.
- [22] I. GHENCIU, *The  $p$ -Gelfand-Phillips property in spaces of operators and Dunford-Pettis type like sets*, Acta Math. Hungar. **155**, (2018) 439–457.
- [23] I. GHENCIU, *Some classes of Banach spaces and complemented subspaces of operators*, Adv. Oper. Theory, **4** (2019), 369–387.
- [24] M. L. LOURENCO AND V. C. C. MIRANDA, *The property (d) and almost limited completely continuous operators*, Analysis Math. **49** (2023), 225–241.
- [25] T. LEAVELLE, *The Reciprocal Dunford-Pettis and Radon-Nikodym properties in Banach lattices*, Ph.D. Thesis, 1984.
- [26] N. MACHRAFI, A. ELBOUR AND M. MOUSSA, *Some characterizations of almost limited sets and applications*, arXiv:1312.2770v1.
- [27] P. MEYER-NIEBERG, *Banach Lattices*, Universitext, Springer-Verlag, Berlin, 1991.
- [28] C. PALAZUELOS, E. A. SANCHEZ PEREZ AND P. TRADACETE, *Maurey-Rosenthal factorization for  $p$ -summing operators and Dodds-Fremlin domination*, J. Operator Theory. **68** (2012), 205–222.
- [29] M. SALIMI AND S. M. MOSHTAGHIOUN, *A new class of Banach spaces and its relation with some geometric properties of Banach spaces*, Hindawi Publishing Corporation, Abstract and Applied Analysis, 2012.
- [30] M. SALIMI AND S. M. MOSHTAGHIOUN, *The Gelfand-Phillips property in closed subspaces of some operator spaces*, Banach J. Math. Anal. **2** (2011), 84–92.
- [31] J. A. SANCHEZ, *Positive Schur property in Banach lattices*, Extr. Math., **7**, (1992), 161–163.
- [32] A. W. WICKSTEAD, *Converses for the Dodds-Fremlin and Kalton-Saab theorems*, Math. Proc. Camb. Phil. So. **120** (1996), 175–179.
- [33] W. WNUK, *Banach Lattices with Order Continuous Norms*, Polish Scientific Publishers PWN, Warsaw, 1999.
- [34] W. WNUK, *On the dual positive Schur property in Banach lattices*, Positivity **2** (2012), 759–773.
- [35] W. WNUK, *Banach lattices with the weak Dunford–Pettis property*, Atti Semin. Mat. Fis. Univ. Modena **42** (1994), 227–236.
- [36] Y. WEN, JI. CHEN, *Characterizations of Banach spaces with relatively compact Dunford-Pettis sets*, Advances in Mathematics, **45** (2016), 122–132.
- [37] B. XIANG, J. X. CHEN AND L. LI, *Weak precompactness in Banach lattices*, Positivity **26** (2022), Issue 1, Article 4, <https://doi.org/10.1007/s11117-022-00873-6>.
- [38] E. ZEEKOEI, *Classes of Dunford-Pettis-type operators with applications to Banach spaces and Banach lattices*, Ph.D. Thesis, 2017.