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CHARACTERIZATIONS OF CLOSED EP OPERATORS ON HILBERT SPACES

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Abstract. In this paper, we present intriguing findings that characterize both the closed (unbounded) and bounded EP operators on Hilbert spaces. We establish the necessary and sufficient conditions for the product of a closed EP operator and a bounded EP operator to also be EP. Additionally, we demonstrate the result $\gamma(T) \leqslant r(T)$, where T is a bounded EP operator, and $\gamma(T)$ and $r(T)$ represent the reduced minimum modulus and the spectral radius of T, respectively.

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