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SCHATTEN p–NORM AND EUCLIDEAN OPERATOR RADIUS INEQUALITIES

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Abstract. From the positivity of 2×2 block matrices, we study the Schatten p-norm inequalities for the sum and product of $n\times n$ complex matrices via the singular values and the Moore-Penrose inverses. Using these inequalities, we deduce the p-numerical radius and the classical numerical radius bounds. We also study the Euclidean operator radius inequalities for a pair of bounded linear operators via the Moore-Penrose inverse, and we deduce new classical numerical radius bounds.

Mathematics subject classification (2020): Primary 47A12; Secondary 15A60, 47A30, 47A50.
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Paper OaM-20-09

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