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GENERALIZED NUMERICAL RADIUS INEQUALITIES FOR CERTAIN OPERATOR MATRICES

SATYAJIT SAHOO* AND NARAYAN BEHERA

Abstract. In this article, a series of new inequalities involving the q-numerical radius for $n \times n$ tridiagonal, and anti-tridiagonal operator matrices has been established. These inequalities serve to establish both lower and upper bounds for the q-numerical radius of operator matrices. Additionally, we developed q-numerical radius inequalities for $n \times n$ circulant, skew circulant, imaginary circulant, and imaginary skew circulant operator matrices. Important examples have been used to illustrate the developed inequalities. In this regard, analytical expressions and a numerical algorithm have also been employed to obtain the q-numerical radii. This comprehensive analysis provides valuable insights into q-numerical radius inequalities shedding light on the intricate relationships within the realm of operators and their matrix representations. We also provide a concluding section, which may lead to several new problems in this area.

Mathematics subject classification (2020): 47A12, 47A30, 47A08, 47A63, 15A60.
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Paper OaM-20-11

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