Differential
Equations
& Applications
Volume 1, Number 4 (2009), 497–515

MULTIPLICITY OF 2–NODAL SOLUTIONS FOR A SEMILINEAR ELLIPTIC EQUATION

TSUNG-FANG WU

Abstract. In this paper, we consider the multiplicity of 2-nodal solutions of semilinear elliptic equations. Using the generalized barycenter map, we prove that existence of multiple 2-nodal solutions for semilinear elliptic equations in some domains with hole.

Mathematics subject classification (2000): 35J60, 35J20, 35J25.
Keywords and phrases: semilinear elliptic equations, multiple nodal solutions, Palais-Smale.

REFERENCES

[1] A. AMBROSETTI AND P. H. RABINOWITZ, *Dual variational methods in critical point theory and applications*, J. Funct. Anal., **14** (1973), 349–381.
[2] A. BAHRI AND P. L. LIONS, *On the existence of positive solutions of semilinear elliptic equations in unbounded domains*, Ann. Inst. H. Poincaré Anal. Non Lineairé, **14**, 3 (1997), 365–413.
[3] T. BARTSCH, *Critical point theory in partially ordered Hilbert spaces*, J. Funct. Anal., **186** (2001), 117–152.
[4] T. BARTSCH AND T. WETH, *Three nodal solutions of singularly perturbed elliptic equations on domains without topology*, Ann. Inst. H. Poincaré Anal. Non Linéaire, **22** (2005), 259–281.
[5] V. BENCI AND G. CERAMI, *Positive solutions of some nonlinear elliptic equations in exterior domain*, Arch. Ration. Mech. Anal., **99** (1987), 283–300.
[6] H. BERESTYCKI AND P. L. LIONS, *Nonlinear scalar field equations. I. Existence of ground state*, Arch. Ration. Mech. Anal., **82** (1983), 313–345.
[7] H. BRÉZIS AND E. H. LIEB, *A relation between pointwise convergence of functions and convergence of functionals*, Proc. Amer Math. Soc., **88** (1983), 486–490.
[8] A. CASTRO AND M. CLAPP, *The effect of the domain topology on the number of minimal nodal solutions of an elliptic equation at critical growth in a symmetric domain*, Nonlinearity, **16** (2003), 579–590.
[9] K. C. CHEN, K. J. CHEN AND H. C. WANG, *Symmetry of positive solutions of semilinear elliptic equations on infinite strip domains*, J. Differential Equations, **148** (1998), 1–8.
[10] G. CERAMI AND D. PASSASEO, *The effect of concentrating potentials in some singularly perturbed problems*, Calc. Var., **17** (2003), 257–281.
[11] M. CLAPP AND T. WETH, *Minimal nodal solutions of the pure critical exponent problem on a symmetric doamin*, Calc. Var. Partial Differential Equations, **21** (2004), 1–14.
[12] M. DEL PINO AND P. L. FELMER, *Local mountain passes for semilinear elliptic problems in unbounded domains*, Calc. Var. Partial Differential Equations, **4** (1996), 121–137.
[13] M. DEL PINO AND P. L. FELMER, *Least energy solutions for elliptic equations in unbounded domains*, Proc. Roy. Soc. Edinburgh Sect. A, **126** (1996), 195–208.
[14] M. J. ESTEBAN AND P. L. LIONS, *Existence and non-existence results for semilinear elliptic problems in unbounded domains*, Proc. Roy. Soc. Edinburgh Sect. A, **93** (1982), 1–12.
[15] M. F. FURTADO, *A relation between the domain topology and the number of minimal nodal solutions for a quasilinear elliptic problem*, Nonlinear Analysis, **62** (2005), 615–628.
[16] M. F. FURTADO, *A note on the number of nodal solutions of an elliptic equation with symmetry*, Applied Mathematics Letters, **19** (2006), 326–331.

© ELEMENT, Zagreb
Paper DEA-01-28

[17] H. C. HUANG AND T. F. WU, *Four 2–nodal solutions for a semilinear elliptic equation in a finite strip with a hole*, J. Math. Anal. Appl., **328** (2007), 567–576.

[18] M. K. KWONG, *Uniqueness of positive solution of* $\Delta u - u + u^p = 0$ *in* $\mathbb{R}^N$, Arch. Rat. Math. Anal., **105** (1989), 243–266.

[19] W. C. LIEN, S. Y. TZENG AND H. C. WANG, *Existence of solutions of semilinear elliptic problems in unbounded domains*, Differential Integral Equations, **6** (1993), 1281–1298.

[20] P. L. LIONS, *The concentration-compactness principle in the calculus of variations. The local compact case*, Ann. Inst. H. Poincaré Anal. Non Lineairé, **1** (1984), 102–145 and 223–283.

[21] Z. Q. WANG, *On a superlinear elliptic equation*, Ann. Inst. H. Poincaré Anal. Non Linéaire, **8** (1991), 43–57.

[22] H. C. WANG AND T. F. WU, *Symmetry breaking in a bounded symmetry domain*, NoDEA-Nonlinear Differential Equations Appl., **11** (2004), 361–377.

[23] M. WILLEM, *Minimax Theorems*, Birkhäuser, Boston, 1996.

[24] T. F. WU, *Multiple positive solutions for semilinear elliptic equations in Esteban-Lions domains with holes*, Taiwanese Journal of Math., **9**, 2 (2005), 245–260.

[25] T. F. WU, *The effect of domain shape on the number of positive and nodal solutions for semilinear elliptic equations*, Nonlinear Analysis T.M.A., **67** (2007), 2609–2622.

[26] T. F. WU, *Existence and multiplicity of nodal solutions for Dirichlet problems in upper half strip with holes*, Nonlinear Analysis T.M.A., **69** (2008), 2167–2178.

[27] E. ZEIDLER, *Nonlinear functional analysis and it Applications I, Fixed-point theorems*, Springer, New Yourk, 1986.