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HOMOCLINIC SOLUTIONS FOR A CLASS OF SECOND ORDER HAMILTONIAN SYSTEMS

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Abstract. The existence of homoclinic solutions is obtained by the Mountain Pass theorem for a class of the second order Hamiltonian systems $\ddot{q}(t)+\nabla V(t,q(t))=0$, where $V(t,x)=-K(t,x)+W(t,x)\in C^1(\mathbb{R}\times\mathbb{R}^N,\mathbb{R})$, $K(t,x)$ is not a quadratic form in x and $W(t,x)$ is superquadratic in x.

1. Introduction and main results

In this paper we shall study the existence of homoclinic orbits for the second order Hamiltonian systems of the type:

$$\ddot{q}(t)+\nabla V(t,q(t))=0, \tag{1}$$

where $V\in C^1(\mathbb{R}\times\mathbb{R}^N,\mathbb{R})$. We say that a solution q is a homoclinic (to 0) if $q\in C^2(\mathbb{R},\mathbb{R}^N)$, $q(t)\to 0$ as $|t|\to\infty$(see [6]).

The existence and multiplicity of homoclinic solutions for Hamiltonian systems have been extensively investigated in many recent papers, see, e.g., ([1–9], [11-20]). But except for [6] and [16], most of the known results on problem (1) are obtained under the following assumption that

$$V(t,x)=-\frac{1}{2}(L(t)x,x)+W(t,x),$$

where $L(t)$ is a symmetric matrix valued function for all $t\in\mathbb{R}$. The main feature of the problem is the lack of global compactness due to unboundedness of domain. To overcome the difficulty, many authors have considered the periodic case, autonomous case or asymptotically periodic case (see [1], [2], [3], [5], [6], [13], [14], [17]). Some papers treat the symmetric case (see [7], [9]). Recently, a coercive condition on L is introduced, that is,

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(L) the $L(t)$ is a positive definite symmetric matrix for $t \in \mathbb{R}$ and there exists an $l \in C(\mathbb{R},(0,\infty))$ such that $l(t) \to +\infty$ as $|t| \to \infty$ and

$$(L(t)x,x) \geqslant l(t)|x|^2 \text{ for all } t \in \mathbb{R} \text{ and } x \in \mathbb{R}^N,$$

(see [4], [8], [11], [12], [14], [15], [16], [18], [19], [20]). For example, in [8], the authors obtained the existence of homoclinic orbits of problem (1) under a class of new superquadratic conditions, which is the following theorem.

THEOREM A. (see [8]) Assume that

$$V(t,x) = -\frac{1}{2}(L(t)x,x) + W(t,x),$$

where $L(t)$ satisfies (L), and the following conditions hold:
(H_1) $W(t,0) \equiv 0$, $W \in C^1(\mathbb{R} \times \mathbb{R}^N, \mathbb{R})$ and $|\nabla W(t,x)| = o(|x|)$ as $|x| \to 0$ uniformly in $t \in \mathbb{R}$;
(H_2) there are two constants $\mu > 2$ and $\nu \in \left[0, \frac{\mu}{2} - 1\right)$ and $\beta \in L^1(\mathbb{R}, \mathbb{R}^+)$ such that

$$(\nabla W(t,x),x) - \mu W(t,x) \geqslant -\nu(L(t)x,x) - \beta(t)$$

for all $t \in \mathbb{R}$ and $x \in \mathbb{R}^N \setminus \{0\}$;
(H_3) there exists $T_0 > 0$ such that

$$\liminf_{|x|\to\infty} \frac{W(t,x)}{|x|^2} > \frac{\pi^2}{2T_0^2} + \frac{l_1}{2}$$

uniformly in $t \in [-T_0, T_0]$, where l_1 is the biggest eigenvalue of $L(t)$ on $[-T_0, T_0]$. Then (1) has at least one nontrivial homoclinic solution. □

In [16], the authors consider that

$$V(t,x) = -K(t,x) + W(t,x),$$

where K is not necessarily homogeneous of degree 2 with respect to x and W satisfies the (AR) condition. Motivated by this paper and [8], we consider the case that K is not necessarily homogeneous of degree 2 with respect to x and W satisfies (H_2). An existence theorem is obtained for homoclinic solutions by applying the Mountain Pass theorem. The main result is the following theorem.

THEOREM 1. *Assume that*

$$V(t,x) = -K(t,x) + W(t,x),$$

where W satisfies (H_1), and the following conditions hold:
(K_1) $K \in C^1(\mathbb{R} \times \mathbb{R}^N, \mathbb{R})$ and there exists a positive constant λ such that

$$\frac{\lambda}{2}(L(t)x,x) \geqslant K(t,x) \geqslant \frac{1}{2}(L(t)x,x), \ \forall t \in \mathbb{R}, \ \forall x \in \mathbb{R}^N,$$

where $L(t)$ *is a positive definite symmetric matrix valued function for all* $t \in \mathbb{R}$*;*
(K_2*)* $\frac{K(t,x)}{|x|^2} \to +\infty$ *as* $|t| \to \infty$ *uniformly in* $x \in \mathbb{R}^N \setminus \{0\}$*;*
(K_3*) there exists a constant* $c_0 > 0$ *such that*

$$0 \leqslant 2K(t,x) - (\nabla K(t,x),x) \leqslant c_0|x|^2,\ \forall t \in \mathbb{R},\ \forall x \in \mathbb{R}^N;$$

(W_1*) there exist constants* $\mu > 2$*,* $\nu \in [0, \frac{\mu}{2\lambda} - \frac{1}{\lambda})$ *and* $\beta(t) \in L^1(\mathbb{R}, \mathbb{R}^+)$ *such that*

$$(\nabla W(t,x),x) - \mu W(t,x) \geqslant -\nu(\nabla K(t,x),x) - \beta(t),\ \forall t \in \mathbb{R},\ \forall x \in \mathbb{R}^N \setminus \{0\};$$

(W_2*) there exists* $T_0 > 0$ *such that*

$$\liminf_{|x|\to\infty} \frac{W(t,x)}{|x|^2} > \frac{\pi^2}{2{T_0}^2} + \frac{\lambda l_1}{2},$$

uniformly in $t \in [-T_0, T_0]$*, where* l_1 *is the biggest eigenvalue of* $L(t)$ *on* $[-T_0, T_0]$*. Then* (1) *has at least one nontrivial homoclinic solution.*

REMARK 1. On one hand, noting that conditions (K_1)-(K_3) can be satisfied if $K(t,x) = \frac{1}{2}(L(t)x,x)$, where $L(t)$ satisfying (L). On the other hand, we can check that if

$$K(t,x) = \Big(2 + t^2 - \frac{1}{|x|^2+1}\Big)|x|^2 \quad \text{and} \quad W(t,x) = |x|^4,$$

where $t \in \mathbb{R}$, $x \in \mathbb{R}^N$, then

$$V(t,x) = -K(t,x) + W(t,x)$$

can not be represented in the form

$$V_1(t,x) = -\frac{1}{2}(L(t)x,x) + W_1(t,x)$$

with $L(t)$ satisfying (L) and W_1 satisfying (H_1)-(H_3). So we extend the results in Theorem 1 of [8]. Besides, (W_1) is different from the (AR) condition used in [16] and conditions (W_4) and (W_5) used in [18], thus our result is different from results in [16] and [18]. There isn't any periodic assumption on W, so our result is also different from Theorem 1.3 in [17].

REMARK 2. In [8], the homoclinic orbit is obtained as a limit of solutions of a certain sequence of boundary-value problems which are obtained by the minimax methods. However, we get the the existence of homoclinic solutions only by applying the Mountain Pass theorem. Moreover, $(L(t)x,x)$ is homogeneous of degree 2 with respect to x, which property plays an important role in the proofs of [8]. For example, it can be included into the norm and be well controlled, that is,

$$\Big(\int_{\mathbb{R}} [|\dot{q}|^2 + (L(t)q,q)]dt\Big)^{\frac{1}{2}}$$

is a norm. However, $K(t,x)$ here is not necessarily homogeneous of degree 2 with respect to x, so

$$\left(\int_{\mathbb{R}}[|\dot{q}|^2+K(t,q)]dt\right)^{\frac{1}{2}}$$

is not a norm in general. Hence we need other conditions, such as (K_1) and (K_3), to overcome the difficulty.

2. Proof of the main results

Let

$$E=\left\{q\in H^1(\mathbb{R},\mathbb{R}^N)\Big|\int_{\mathbb{R}}[\,|\dot{q}|^2+(L(t)q,q)\,]dt<+\infty\right\}.$$

Then E is a Hilbert space with the norm given by

$$\|q\|=\left(\int_{\mathbb{R}}[\,|\dot{q}|^2+(L(t)q,q)\,]dt\right)^{\frac{1}{2}}.$$

Obviously, by (K_1) E is continuously embedded in $H^1(\mathbb{R},\mathbb{R}^N)$ and so continuously embedded in $L^p(\mathbb{R},\mathbb{R}^N)$ for $p\in[2,\infty]$. Thus we have

$$\|q\|_{L^p}\leqslant\gamma_p\|q\|\ \text{ for } p\in[2,\infty],$$

where $\gamma_p>0$. For $q\in E$, let

$$I(q)=\frac{1}{2}\int_{\mathbb{R}}[\,|\dot{q}|^2+2K(t,q)\,]dt-\int_{\mathbb{R}}W(t,q)dt.$$

Then $I\in C^1(E,\mathbb{R})$ and it is routine to verify that any critical point of I on E is a classical solution of (1) with $q(\pm\infty)=0$.

LEMMA 1. (see [4]) *Suppose $L(t)$ satisfies the condition (L), then the imbedding of E in $L^2(\mathbb{R},\mathbb{R}^N)$ and $L^\infty(\mathbb{R},\mathbb{R}^N)$ are compact.*

LEMMA 2. *Suppose K satisfies the conditions (K_1) and (K_2), then the imbedding of E in $L^2(\mathbb{R},\mathbb{R}^N)$ and $L^\infty(\mathbb{R},\mathbb{R}^N)$ are compact.*

Proof. It is obvious that by (K_1) and (K_2) one can obtain (L). Then by Lemma 1, the lemma is proved. □

PROOF OF THEOREM 1. We divide our proof into four steps.

Step 1. There are $\rho>0,\ \delta>0$ such that

$$I|_S\geqslant\delta,$$

where $S = \{q \in E \mid \|q\| = \rho\}$. In fact, it follows from (H_1) that, for any $\varepsilon > 0$, there exists $\rho' = \rho'(\varepsilon) > 0$ such that

$$|\nabla W(t,x)| \leqslant 2\varepsilon|x|, \quad \forall\, |x| \leqslant \rho', \forall t \in \mathbb{R},$$

which implies that

$$|W(t,x)| \leqslant \varepsilon|x|^2$$

for all $|x| \leqslant \rho'$ and for all $t \in \mathbb{R}$. Then choose $\varepsilon = (4\gamma_2^2)^{-1} > 0$, $\rho = \rho'/\gamma_\infty > 0$ and $\delta = \rho^2/4 > 0$, and by (K_1) we have

$$\begin{aligned} I(q) &= \frac{1}{2}\int_{\mathbb{R}} [|\dot{q}|^2 + 2K(t,q)]dt - \int_{\mathbb{R}} W(t,q)dt \\ &\geqslant \frac{1}{2}\|q\|^2 - \int_{\mathbb{R}} W(t,q)dt \\ &\geqslant \frac{1}{2}\|q\|^2 - \varepsilon\int_{\mathbb{R}} |q|^2 dt \\ &\geqslant \frac{1}{2}\|q\|^2 - \varepsilon\gamma_2^2\|q\|^2 \\ &= \frac{1}{4}\|q\|^2 = \delta \end{aligned}$$

for all $q \in S$.

Step 2. By (W_2), there exist constants $\varepsilon_1 > 0$ and $\sigma > 0$ such that

$$\frac{W(t,x)}{|x|^2} \geqslant \frac{\pi^2 + \varepsilon_1}{2T_0^2} + \frac{\lambda l_1}{2}$$

for all $|x| > \sigma$ and $t \in [-T_0, T_0]$. Let

$$\delta_1 = \max_{t\in[-T_0,T_0],\ |x|\leqslant\sigma} |W(t,x)|,$$

then we have

$$W(t,x) \geqslant \left(\frac{\pi^2 + \varepsilon_1}{2T_0^2} + \frac{\lambda l_1}{2}\right)(|x|^2 - \sigma^2) - \delta_1 \tag{2}$$

for all $x \in \mathbb{R}$ and $t \in [-T_0, T_0]$. Let

$$e(t) = \begin{cases} \xi|\sin(\omega t)|e_1, & t \in [-T_0, T_0], \\ 0, & t \in \mathbb{R}\setminus[-T_0, T_0], \end{cases}$$

where $\xi \in \mathbb{R}$, $\omega = \frac{\pi}{T_0}$ and $e_1 = (1, 0, \cdots, 0)$. By (K_1), (W_2) and (2) one has

$$\begin{aligned} I(e) &\leqslant \frac{1}{2}\int_{\mathbb{R}} |\dot{e}(t)|^2 dt + \frac{\lambda}{2}\int_{\mathbb{R}} (L(t)e(t), e(t))dt - \int_{\mathbb{R}} W(t,e(t))dt \\ &= \frac{1}{2}\xi^2\omega^2\int_{-T_0}^{T_0} |\cos(\omega t)|^2 dt + \frac{\lambda}{2}\int_{-T_0}^{T_0} (L(t)e(t), e(t))dt \end{aligned}$$

$$
\begin{aligned}
&\quad - \int_{-T_0}^{T_0} W(t,\xi|\sin(\omega t)|e_1)dt \\
&\leqslant \frac{1}{2}\xi^2\omega^2 \int_{-T_0}^{T_0} |\cos(\omega t)|^2 dt + \frac{\lambda l_1}{2}\xi^2 \int_{-T_0}^{T_0} |\sin(\omega t)|^2 dt \\
&\quad - \Big(\frac{\pi^2+\varepsilon_1}{2T_0^2} + \frac{\lambda l_1}{2}\Big)\xi^2 \int_{-T_0}^{T_0} |\sin(\omega t)|^2 dt + 2T_0\left[\Big(\frac{\pi^2+\varepsilon_1}{2T_0^2} + \frac{\lambda l_1}{2}\Big)\sigma^2 + \delta_1\right] \\
&= \frac{1}{2}\xi^2\omega^2 \int_{-T_0}^{T_0} [|\cos(\omega t)|^2 - |\sin(\omega t)|^2] dt \\
&\quad - \frac{\varepsilon_1\xi^2}{2T_0^2} \int_{-T_0}^{T_0} |\sin(\omega t)|^2 dt + 2T_0\left[\left(\frac{\pi^2+\varepsilon_1}{2T_0^2} + \frac{\lambda l_1}{2}\right)\sigma^2 + \delta_1\right] \\
&= -\frac{\varepsilon_1}{2T_0}\xi^2 + 2T_0\left[\left(\frac{\pi^2+\varepsilon_1}{2T_0^2} + \frac{\lambda l_1}{2}\right)\sigma^2 + \delta_1\right] \to -\infty \ \text{ as } \xi \to \infty.
\end{aligned}
$$

We can choose a large enough ξ such that $\|e\| \geqslant \rho$ and $I(e) < 0$.

Step 3. Let $\{q_k\} \subset E$ be a (PS) sequence, i.e., there exists a constant $M_1 > 0$ such that

$$
|I(q_k)| \leqslant M_1 \quad \text{for all } k \text{ and } I'(q_k) \to 0 \quad \text{as } k \to \infty. \tag{3}
$$

By (K_3), we have

$$
\begin{aligned}
\frac{2}{\mu}\langle I'(q_k), q_k\rangle &= \frac{2}{\mu}\int_{\mathbb{R}} [|\dot{q}_k|^2 + (\nabla K(t,q_k), q_k) - (\nabla W(t,q_k), q_k)]dt \\
&\leqslant \frac{2}{\mu}\int_{\mathbb{R}} [|\dot{q}_k|^2 + 2K(t,q_k) - (\nabla W(t,q_k), q_k)]dt.
\end{aligned}
$$

Then one gets

$$
-\frac{2}{\mu}\int_{\mathbb{R}} [|\dot{q}_k|^2 + 2K(t,q_k)]dt \leqslant -\frac{2}{\mu}\langle I'(q_k), q_k\rangle - \frac{2}{\mu}\int_{\mathbb{R}} (\nabla W(t,q_k), q_k)dt. \tag{4}
$$

From (W_1) one obtains

$$
\begin{aligned}
&\int_{\mathbb{R}} [|\dot{q}_k|^2 + 2K(t,q_k)]dt \\
&\qquad = 2I(q_k) + 2\int_{\mathbb{R}} W(t,q_k)dt \\
&\qquad \leqslant 2I(q_k) + \frac{2}{\mu}\int_{\mathbb{R}} [(\nabla W(t,q_k), q_k) + \nu(\nabla K(t,q_k), q_k) + \beta(t)]dt.
\end{aligned} \tag{5}
$$

From (4) and (5), we have

$$
\begin{aligned}
\left(1 - \frac{2}{\mu}\right)\int_{\mathbb{R}} [|\dot{q}_k|^2 + 2K(t,q_k)]dt &\leqslant 2I(q_k) - \frac{2}{\mu}\langle I'(q_k), q_k\rangle \\
&\quad + \frac{2\nu}{\mu}\int_{\mathbb{R}} (\nabla K(t,q_k), q_k)dt + \frac{2}{\mu}\int_{\mathbb{R}} \beta(t)dt.
\end{aligned}
$$

Then by (K_1), (K_3) and (3) one has

$$\begin{aligned}\left(1-\frac{2}{\mu}\right)\int_{\mathbb{R}}[|\dot{q}_k|^2dt+(L(t)q_k,q_k)]dt \\ &\leqslant 2I(q_k)-\frac{2}{\mu}\langle I'(q_k),q_k\rangle \\ &\quad+\frac{4}{\mu}\nu\int_{\mathbb{R}}K(t,q_k)dt+\frac{2}{\mu}\int_{\mathbb{R}}\beta(t)dt \\ &\leqslant 2M_1+\frac{2}{\mu}\|I'(q_k)\|\|q_k\| \\ &\quad+\frac{2\lambda\nu}{\mu}\int_{\mathbb{R}}(L(t)q_k,q_k)dt+\frac{2}{\mu}\|\beta\|_{L^1(\mathbb{R},\mathbb{R}^+)},\end{aligned}$$

that is,

$$\begin{aligned}\left(1-\frac{2}{\mu}\right)\int_{\mathbb{R}}|\dot{q}_k|^2dt+\left(1-\frac{2}{\mu}-\frac{2\lambda\nu}{\mu}\right)\int_{\mathbb{R}}(L(t)q_k,q_k)dt \\ \leqslant 2M_1+\frac{2}{\mu}\|I'(q_k)\|\|q_k\|+\frac{2}{\mu}\|\beta\|_{L^1(\mathbb{R},\mathbb{R}^+)}.\end{aligned} \tag{6}$$

Since $\mu>2$ and $\nu\in[0,\frac{\mu}{2\lambda}-\frac{1}{\lambda})$, (6) shows that $\{q_k\}_{k\in N}$ is bounded in E.

Step 4. Since $\{q_k\}$ is bounded, that is, there exists a positive constant C_1 such that

$$\|q_k\|\leqslant C_1 \tag{7}$$

for all $k\in N$. Passing to a subsequence if necessary, by Lemma 2, we may assume that

$$q_k\rightharpoonup q \quad \text{in } E \tag{8}$$

and

$$q_k\to q \ \text{ in } L^2(\mathbb{R},\mathbb{R}^N) \text{ and } L^\infty(\mathbb{R},\mathbb{R}^N) \tag{9}$$

as $k\to\infty$. In addition, it follows from (H_1) that there is $\sigma_1>0$ such that

$$|\nabla W(t,x)|\leqslant|x| \tag{10}$$

for all $t\in\mathbb{R}$ and all $|x|\leqslant\sigma_1$. Since E is continuously embedded in $H^1(\mathbb{R},\mathbb{R}^N)$, one has

$$|q(t)|\to 0 \text{ as } |t|\to\infty. \tag{11}$$

From (11) and (9) it follows that there exists $\delta_2>0$ such that

$$|q_k(t)|\leqslant\sigma_1 \tag{12}$$

for k large and all $|t|>\delta_2$. Thus, by (12), (10), (7) and Hölder's inequality, for k large one has

$$\left|\int_{\mathbb{R}}(\nabla W(t,q_k)-\nabla W(t,q),q_k-q)dt\right|$$

$$
\begin{aligned}
&\leqslant \int_{\mathbb{R}} (|\nabla W(t,q_k)| + |\nabla W(t,q)|)|q_k - q|dt \\
&\leqslant \int_{|t|>\delta_2} (|q_k| + |q|)|q_k - q|dt + 2C_2 \int_{|t|\leqslant\delta_2} |q_k - q|dt \\
&\leqslant [\|q_k\|_{L^2} + \|q\|_{L^2} + 2C_2(2\delta_2)^{\frac{1}{2}}]\|q_k - q\|_{L^2} \\
&\leqslant [\gamma_2\|q_k\| + \|q\|_{L^2} + 2C_2(2\delta_2)^{\frac{1}{2}}]\|q_k - q\|_{L^2} \\
&\leqslant C_3\|q_k - q\|_{L^2}, \qquad (13)
\end{aligned}
$$

where

$$C_2 = \max\{|\nabla W(t,x)| \mid |t| \leqslant \delta_2, |x| \leqslant \gamma_\infty C_1\}$$

and

$$C_3 = \gamma_2 C_1 + \|q\|_{L^2} + 2C_2(2\delta_2)^{\frac{1}{2}}.$$

By (K_3) we have

$$
\begin{aligned}
\langle I'(q), q\rangle &= \int_{\mathbb{R}} [|\dot{q}|^2 + (\nabla K(t,q), q) - (\nabla W(t,q), q)]dt \\
&\geqslant \int_{\mathbb{R}} [|\dot{q}|^2 + 2K(t,q) - c_0|q|^2 - (\nabla W(t,q), q)]dt.
\end{aligned}
$$

Then by (K_1) and (13) one has

$$
\begin{aligned}
\|q_k - q\|^2 &\leqslant \langle I'(q_k) - I'(q), q_k - q\rangle + c_0\|q_k - q\|_{L^2}^2 \\
&\quad + \int_{\mathbb{R}} (\nabla W(t,q_k) - \nabla W(t,q), q_k - q)\, dt \\
&\leqslant \|I'(q_k)\|\|q_k - q\| - \langle I'(q), q_k - q\rangle + c_0\|q_k - q\|_{L^2}^2 + C_3\|q_k - q\|_{L^2} \\
&\leqslant \|I'(q_k)\|(C_1 + \|q\|) - \langle I'(q), q_k - q\rangle + c_0\|q_k - q\|_{L^2}^2 + C_3\|q_k - q\|_{L^2} \qquad (14)
\end{aligned}
$$

for k large. From (14), (3), (8) and (9) it follows that $q_k \to q$ in E as $k \to \infty$. Thus I satisfies the (PS) condition. Then by the Mountain Pass theorem (see [10]), q is the nontrivial homoclinic orbit of problem (1). The proof is complete. □

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