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OSCILLATION AND NONOSCILLATION CRITERIA FOR EVEN ORDER NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract. This paper is devoted to the study of the oscillatory and nonoscillatory behavior of even order nonlinear functional differential equations with deviating argument of the type

$$\left(p(t)|x^{(n)}(t)|^{\alpha}\operatorname{sgn}x^{(n)}(t)\right)^{(n)}+q(t)|x(g(t))|^{\beta}\operatorname{sgn}x(g(t))=0. \qquad (\mathrm{A_g})$$

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