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MATCHING METHOD FOR NODAL SOLUTIONS OF MULTI–POINT BOUNDARY VALUE PROBLEMS

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Abstract. In this paper, we study the nonlinear boundary value problem consisting of the equation $y'' + w(t)f(y) = 0$ on $[a,b]$ and two multi-point boundary conditions. We establish the existence of various nodal solutions of this problem by matching the solutions of two boundary value problems, each of which involves one separated boundary condition and one multi-point boundary condition, at some point in (a,b). We also obtain conditions for this problem not to have certain types of nodal solutions.

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