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SECOND–ORDER DIFFERENTIAL EQUATIONS: SOME SIGNIFICANT RESULTS DUE TO JAMES S.W. WONG

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Abstract. Wong's contribution in the qualitative theory of second-order differential equations is well-known to a large mathematical auidence. Among a huge number of published Wong's papers, in this survey article, we analyze only a few Wong's theorems including their consequences, examples and many influences to other mathematicians dealing with oscillations of second-order differential and functional differential equations as well as of corresponding dynamic equations on time scales.

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