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DIFFUSIVE SOLUTIONS OF THE COMPETITIVE LOTKA–VOLTERRA SYSTEM

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Abstract. In the present work we show by means of explicit construction that three new types of solutions exist for the one dimensional competitive Lotka-Volterra reaction-diffusion system. The new solutions constructed are (i) space-time separated solutions, (ii) unbounded solutions, and (iii) solutions of Gaussian type, with the constructions being based largely on the standard methods for constructing solutions to the one-dimensional heat equation. From these exact solutions a new and interesting phenomena is found, namely diffusion-induced long-term coexistence of three species. In addition, the approach to constructing explicit solutions presented here can readily be applied to other reaction-diffusion systems.

Mathematics subject classification (2010): 35K55, 35K57, 35K60.
Keywords and phrases: time-periodic unbounded solution, solution of Gaussian type, Lotka-Volterra, explicit solution, heat equation, reaction-diffusion system.

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