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MULTIPLE POSITIVE SOLUTIONS FOR NONLOCAL BOUNDARY VALUE PROBLEMS WITH p–LAPLACIAN OPERATOR

SHENG-PING WANG

Abstract. The main goal of this article is to establish an existence result for the following multi-point boundary value problem:

$$(\phi_p(u'))' + q(t)f(t,u(t),u'(t)) = 0, \ \ t \in (0,1),$$

$$u(0) = \Sigma_{i=1}^{m-2} a_i u(\xi_i), \ \phi_p(u'(1)) = \Sigma_{i=1}^{m-2} b_i \phi_p(u'(\xi_i)),$$

where $\phi_p(s) = |s|^{p-2}s$, $p > 1$, and $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$. By means of fixed point theorem due to Avery and Peterson, we study the existence of at least three positive solutions to our problem and get some information about these solutions under some sufficient conditions posed.

Mathematics subject classification (2010): 34B10, 34B15, 34B18.
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