Differential
Equations
& Applications
Volume 10, Number 1 (2018), 75–86 doi:10.7153/dea-2018-10-05

A CONNECTION BETWEEN REGULARITY AND DIRICHLET PROBLEMS FOR NON–DIVERGENCE ELLIPTIC EQUATIONS

JORGE RIVERA-NORIEGA

Abstract. We observe that a version of Poincaré's inequality for positive solutions to second order linear non-divergence form equations vanishing on a portion of the boundary, implies a natural connection between L^p Dirichlet and L^q Regularity problems for this type of equations.

Mathematics subject classification (2010): 35B09,35J25.
Keywords and phrases: Poincaré inequality, L^q Regularity problem, L^p Dirichlet problem, Reverse Hölder inequality.

REFERENCES

[1] P. Bauman, *Positive solutions of elliptic equations in nondivergence form and their adjoints*, Ark. Mat **22** (1984), 153–173.
[2] B. E. J. Dahlberg, *On estimates of harmonic measure*, Arch. Rat. Mech. Anal. **65** (1977), 272–288.
[3] ———, *On the Poisson integral for Lipschitz and C^1 domains*, Studia Math. **66** (1979), 13–24.
[4] ———, *On the absolute continuity of elliptic measure*, Amer. J. of Math. **108** (1986), 1119–1138.
[5] M. Dindos, C. E. Kenig, and J. Pipher, *BMO solvability and the A_∞ condition for elliptic operators*, J. Geom. Anal. **21** (2011), 78–95.
[6] M. Dindos and J. Kirsch, *The regularity problem for elliptic operators with boundary data in Hardy-Sobolev space HS^1*, Math. Res. Lett. **19** (2013), 699–717.
[7] M. Dindos, S. Petermichl, and J. Pipher, *The L^p dirichlet problem for second order elliptic operators and a p-adapted square function*, J. Funct. Anal. **249** (2007), 372–392.
[8] ———, *BMO solvability and the A_∞ condition for second order parabolic operators*, Annals de l'Institut Henri Poincare (C) Non Linear Analysis. doi: 10.1016/j.anihpc.2016.09.004 (2016).
[9] M. Dindos and T. Wall, *The L^p Dirichlet problem for second order, non-divergence form operators: solvability and perturbation results*, J. Funct. Anal. **261** (2011), 1753–1774.
[10] L. Escauriaza and C. E. Kenig, *Area Integral estimates for solutions and normalized adjoint solutions to nondivergence form elliptic equations*, Ark. Mat. **31** (1993), 275–296.
[11] R. Fefferman, C. E. Kenig, and J. Pipher, *The theory of weights and the Dirichlet problem for elliptic equations*, Ann. of Math. **134** (1991), 65–124.
[12] S. Hofmann, J. M. Martell, and S. Mayboroda, *Uniform rectifiability, Carleson measure estimates, and approximation of harmonic functions*, Duke Math. J. **165** (2016), 2331–2389.
[13] D. Jerison and C. E. Kenig, *Boundary behavior of harmonic functions in nontangentially accessible domains*, Adv. in Math **146** (1982), 80–147.
[14] C. E. Kenig, *Potential theory of non-divergence form elliptic equations*, Dirichlet forms (Varenna 1992), Springer, Lecture Notes in Mathematics vol. 1563, 1993, pp. 89–128.
[15] C. E. Kenig, B. Kirchheim, J. Pipher, and T. Toro, *Square functions and the A_∞ property of elliptic measures*, J. Geom. Anal. **16** (2016), 2383–2410.
[16] C. E. Kenig, H. Koch, J. Pipher, and T. Toro, *A new approach to absolute continuity of elliptic measure with applications to non-symmetric equations*, Adv. in Math. **153** (2000), 231–298.
[17] C. E. Kenig and J. Pipher, *The Neumann problem for elliptic equations with non-smooth coefficients*, Invent. Math. **113** (1993), 447–509.
[18] ———, *The absolute continuity of elliptic measure revisited*, J. Fourier Anal. Appl. **4** (1998), 463–468.

© ELEMENT, Zagreb
Paper DEA-10-05

[19] ———, *The Dirichlet problem for elliptic equations with drift terms*, Publ. Mat **45** (2001), 199–217.

[20] C. Rios, *The L^p Dirichlet problem and nondivergence harmonic measure*, Trans. Amer. Math. Soc. **355** (2003), 665–687.

[21] ———, *L^p regularity of the Dirichlet problem for elliptic equations with singular drift*, Publ. Mat. **50** (2006), 475–507.

[22] J. Rivera-Noriega, *Absolute continuity of parabolic measure and area integral estimates in non-cylindrical domains*, Indiana Univ. Math. J. **52** (2003), 477–525.

[23] ———, *Perturbation and solvability of initial L^p Dirichlet problems for parabolic equations over non-cylindrical domains*, Canadian J. Math **66** (2014), 429–452.

[24] C. Sbordone and G. Zecca, *The L^p solvability of the Dirichlet problem for planar elliptic equations*, J. Fourier Anal. Appl. **15** (2009), 871–903.

[25] Z. Shen, *A relationship between the Dirichlet and Regularity problems for elliptic equations*, Math. Res. Lett. **14** (2007), 205–213.

[26] P. Sjögren, *On the adjoint of an elliptic linear differential operator and its potential theory*, Ark. Mat **11** (1973), 153–165.

[27] G. Verchota, *Layer Potentials and regularity for the Dirichlet problem for Laplace's equation in Lipschitz domains*, J. Funct. Anal. **59** (1984), 572–611.

[28] M. Wuertz, The Implicit Function Theorem for Lipschitz Functions and Applications, Master's Thesis, University of Missouri-Columbia, May of 2008.