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UNIFORM EXPONENTIAL STABILITY IN THE SENSE OF HYERS AND ULAM FOR PERIODIC TIME VARYING LINEAR SYSTEMS

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Abstract. We prove that the uniform exponential stability of time depended p-periodic system

$$\dot{\Psi}(t) = \Pi(t)\Psi(t), \quad t \in \mathbb{R}_+, \quad \Psi(t) \in \mathbb{C}^n$$

is equivalent to its Hyers–Ulam stability. As a tool, we consider the exact solution of the Cauchy problem

$$\begin{cases} \dot{\Theta}(t) = \Pi(t)\Theta(t) + e^{i\alpha t}\zeta(t), & t \in \mathbb{R}_+ \\ \Theta(0) = \Theta_0 \end{cases}$$

as the approximate solution of $\dot{\Psi}(t) = \Pi(t)\Psi(t)$, $t \in \mathbb{R}_+$, $\Psi(t) \in \mathbb{C}^n$, where α is any real number, $\zeta(t)$ with $\zeta(0) = 0$, is a p-periodic bounded function on the Banach space $\mathscr{S}(\mathbb{R}_+, \mathbb{C}^n)$. More precisely we prove that the system $\dot{\Psi}(t) = \Pi(t)\Psi(t)$, $t \in \mathbb{R}_+$, $\Psi(t) \in \mathbb{C}^n$ is Hyers–Ulam stable if and only if it is exponentially stable. We argue that Hyers-Ulam stability concept is quite significant in realistic problems in numerical analysis and economics.

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