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ON LINEAR AND NONLINEAR FRACTIONAL HADAMARD BOUNDARY VALUE PROBLEMS

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Abstract. We establish new Lyapunov-type inequalities for linear Hadamard fractional differential equations with pointwise boundary conditions. Furthermore, we employ the contraction mapping principle to obtain the criterion of the existence of a unique solution for a nonlinear fractional Hadamard type boundary value problem.

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REFERENCES

[1] B. AHMED, *Sharp estimates for the unique solution of two-point fractional-order boundary value problems*, Appl. Math. Lett. **65** (2017), 77–82.
[2] G. BORG, *On a Liapunoff criterion of stability*, Amer. J. Math. **71** (1949), 67–70.
[3] S. DHAR AND Q. KONG, *Liapunov-type inequalities for third-order half-linear equations and applications to boundary value problems*, Nonlin. Anal. **110** (2014), 170–181.
[4] S. DHAR AND Q. KONG, *Lyapunov-type inequalities for higher order half-linear differential equations*, Appl. Math. Comput. **273** (2016), 114–124.
[5] S. DHAR AND Q. KONG, *Lyapunov-type inequalities for third-order linear differential equations*, Math. Inequal. Appl. **19** (2016), 297–312.
[6] S. DHAR AND Q. KONG, *Lyapunov-type inequalities for odd order linear differential equations*, Electron. J. Differ. Equ. **2016** (2016), no. 243, 1–10.
[7] S. DHAR, Q. KONG AND M. MCCABE, *Fractional boundary value problems and Lyapunov-type inequalities with fractional integral boundary conditions*, Electron. J. Qual. Theory Differ. Equ. **2016**, no. 43, 1–16.
[8] R. A. C. FERREIRA, *On a Lyapunov-type inequality and the zeros of a certain Mittag-Leffler function*, J. Math. Anal. Appl. **412** (2014), 1058–1063.
[9] R. A. C. FERREIRA, *A Lyapunov-type inequality for a fractional boundary value problem*, Fract. Calc. Appl. Anal. **16** (2013), 978–984.
[10] R. A. C. FERREIRA, *Existence and uniqueness of solutions for two-point fractional boundary value problems*, Electron. J. Differ. Equ. **2016** (2016), 1–5.
[11] P. HARTMAN, *Ordinary Differential Equations*, Wiley, New York, 1964, and Birkhauser, Boston 1982.
[12] M. JLELI AND B. SAMET, *Lyapunov-type inequalities for fractional boundary value problems*, Elec. J. Diff. Eq. **2015** (2015), 1–11.
[13] A. A. KILBAS, H. M. SRIVASTAVA AND J. J. TRUJILO, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematical Studies 204, Elsevier, Amsterdam 2006.
[14] W. G. KELLY AND A. C. PETERSON, *The Theory of Differential Equations*, University text, Springer, New York, 2010.
[15] A. M. LIAPUNOV, *Probleme general de la stabilite du mouvement*, Ann. Math Stud. **17** (1947), 203–474.
[16] Q. MA, C. MA AND J. WANG, *A Lyapunov-type inequality for a fractional differential equation with Hadamard derivative*, J. Math. Ineq. **11** (2017), 135–141.

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[17] J. RONG AND C. BAI, *Lyapunov-type inequality for a fractional differential equation with fractional boundary conditions*, Adv. Diff. Eq. 2015, **2015**:82.
[18] A. WINTNER, *On the non-existence of conjugate points*, Amer. J. Math. **73** (1951), 368–380.