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EXISTENCE OF SOLUTIONS TO NONLINEAR LEGENDRE BOUNDARY VALUE PROBLEMS

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Abstract. In this paper, we consider nonlinearly perturbed Legendre differential equations subject to the usual boundary conditions. For such problems we establish sufficient conditions for the existence of solutions and in some cases we provide a qualitative description of solutions depending on a parameter. The results presented depend on the size and limiting behavior of the nonlinearities.

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