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SOLUTIONS FOR THE FRACTIONAL p–LAPLACIAN SYSTEMS WITH SEVERAL CRITICAL SOBOLEV–HARDY TERMS

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Abstract. In this paper, we consider a class of fractional p-Laplacian system with three fractional critical Sobolev-Hardy exponents. By the Ekeland variational principle and the Mountain-Pass theorem, we study the existence and multiplicity of positive solutions to the system.

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