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EXISTENCE OF SOLUTIONS TO NONLINEAR STURM–LIOUVILLE PROBLEMS WITH LARGE NONLINEARITIES

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Abstract. In this paper, we present results which allow us to establish the existence of solutions to nonlinear Sturm-Liouville problems with unbounded nonlinearities. We consider both regular and singular problems. Our main results rely on a variant of the Lyapunov-Schmidt used in conjunction with topological degree theory.

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