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RANDOM NEUTRAL SEMILINEAR DIFFERENTIAL EQUATIONS WITH DELAY

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Abstract. In this paper, we present the existence and uniqueness of a random mild solution of a system of neutral semilinear random differential equations with delay. Also the Lipschitz regularity of the solution is presented. The results are based on random versions of Perov's fixed point theorem. Finally, some examples are given to illustrate our main result.

Mathematics subject classification (2020): 47H10, 47H30, 54H25.
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