

GLOBAL EXISTENCE, UNIQUENESS AND ASYMPTOTIC BEHAVIOR FOR A NONLINEAR VISCOELASTIC PROBLEM WITH INTERNAL DAMPING AND LOGARITHMIC SOURCE TERM

JORGE FERREIRA, MOHAMMAD SHAHROUZI* ,
SERIK E. AITZHANOV, SEBASTIAO CORDEIRO AND DANIEL V. ROCHA

Abstract. This paper is concerned with the existence of global weak solution for a nonlinear viscoelastic problem with internal damping and a logarithmic source term and Dirichlet boundary initial conditions, and with the study of the asymptotic behavior of the problem, involving: a) exponential decay of total energy of solutions for initial data in the set of stability created by the Nehari manifold, b) the exponential growth of the logarithmic source term for negative initial energy. In the existence of global weak solution we employed similar ideas as in the work of S. Cordeiro, J. Ferreira, et al., 2021, where the Faedo-Galerkin method was combined with Aubin-Lions lemmas for the passage to the limit in the nonlinear terms. In the study of the exponential decay of the total energy and in the growth of the logarithmic term of the energy we adapted the perturbed energy methods in a work of Messaoudi & Tatar, 2006 and 2003.

Mathematics subject classification (2020): Primary 35A01, 35B40, 35D30; Secondary 75D10.

Keywords and phrases: Exponential decay, exponential growth, global existence, nonlinear viscoelastic equation, logarithmic source.

REFERENCES

- [1] A. AMBROSETTI, P. H. RABINOWITZ, *Dual variational methods in critical point theory and applications*, Journal of Functional Analysis, **14**, (1973), 349–381.
- [2] A. AMBROSETTI, A. MALCHIODI, *Nonlinear Analysis and Semilinear Elliptic Problems*, Cambridge Studies in Advanced Mathematics 104, Cambridge University Press, Cambridge, 2007.
- [3] P. DRÁBEK, J. MILOTA, *Methods of Nonlinear Analysis*, Applications to Differential Equations, Birkhäuser, Basel, 2007.
- [4] M. STRUWE, *Variational Methods*, Springer-Verlag, Berlin, 1990.
- [5] J. M. BALL, *Remarks on blow up and nonexistence theorems for nonlinear evolutions equations*, Quart. J. Math. Oxford, **28**, (1977), 473–486.
- [6] M. M. CAVALCANTI, V. N. DOMINGOS CAVALCANTI, I. LASIECKA, F. A. FALCÃ NASCIMENTO, *Intrinsic decay rate estimates for the wave equation with competing viscoelastic and frictional dissipative effects*, Discrete & Continuous Dynamical Systems, **19** (7), (2014), 1987–2011.
- [7] M. M. CAVALCANTI, V. N. DOMINGOS CAVALCANTI, J. FERREIRA, *Existence and uniform decay for a non-linear viscoelastic equation with strong damping*, Mathematical Methods in Applied Sciences, **24** (14), (2001), 1043–1053.
- [8] M. M. CAVALCANTI, V. N. DOMINGOS CAVALCANTI, J. S. PRATES FILHO, J. A. SORIANO, *Existence and uniform decay rates for viscoelastic problems with nonlinear boundary damping*, Differential and Integral Equations, **14** (1), (2001), 85–116.
- [9] M. M. CAVALCANTI, V. N. DOMINGOS CAVALCANTI, J. A. SORIANO, *Exponential decay for the solution of semilinear viscoelastic wave equations with localized damping*, Elect. J. Diff. Eqs, **2002** (44), (2002), 1–14.
- [10] S. M. S. CORDEIRO, D. C. PEREIRA, J. FERREIRA, C. A. RAPOSO, *Global solutions and exponential decay to a Klein-Gordon equation of Kirchhoff-Carrier type with strong damping and nonlinear logarithmic source term*, Partial Differential Equations in Applied Mathematics, **3**, (2021).

- [11] C. M. DAFERMOS, *Asymptotic stability in viscoelasticity*, Arch. Rational Mech. Anal, **37**, (1970), 297–308.
- [12] J. L. LIONS, *Quelques méthodes de resolution des problèmes aux limites non lineaires*, Dunod, Paris, (1968).
- [13] L. GROSS, *Logarithmic Sobolev inequalities*, Amer. J. Math., **97** (1975), 1061–1083.
- [14] S. A. MESSAOUDI, *Blow up and global existence in a nonlinear viscoelastic wave equation*, Mathematische Nachrichten, **260** (1), (2003), 58–66.
- [15] J. E. MUÑOZ RIVERA, E. C. LAPA, R. BARETTO, *Decay rates for viscoelastic plates with memory*, Journal of Elasticity, **44**, (1996), 61–87.
- [16] A. SZULKIN, *The method of Nehari manifold revisited*, Progress in Variational Problems: New Trends of Geometric Gradient Flow and Critical Point Theory, (2011), 89–102.
- [17] N. MEZOUA, S. M. BOULAAARAS, A. ALLAHM, *Global existence of solutions for the viscoelastic Kirchhoff equation with logarithmic source terms*, Hindawi Complexity, (2020).
- [18] C. A. RAPOSO, A. P. CATTAI, J. O. RIBEIRO, *Global solution and asymptotic behaviour for a wave equation type p -Laplacian with memory*, Open J. Math. Anal., **2**, (2018), 156–171.
- [19] S. A. MESSAOUDI, N. TATAR, *Global existence and uniform stability of solutions for a quasilinear viscoelastic problem*, Mathematical Methods in the Applied Sciences, **30**, (2007), 665–680.
- [20] S. A. MESSAOUDI, N. TATAR, *Global existence and asymptotic behavior for a nonlinear viscoelastic problem*, Mathematical Sciences Research Journal, (2003).
- [21] Y. YE, *Global existence and asymptotic behavior of solutions for a class of nonlinear degenerate wave equations*, International Journal of Differential Equations, **2007**, (2007), 1–9.
- [22] M. WILLEM, *Minimax theorems (progress in nonlinear differential equations and their applications*, Springer Science & Business Media, 24, (1997).
- [23] E. ZUAZUA, *Exponential decay for the semilinear wave equation with locally distributed damping*, Comm. PDE., **15**, (1990), 205–235.
- [24] Z. NEHARI, *On a class of nonlinear second-order differential equations*, Trans. Amer. Math. Soc. **95**, (1960), 101–123.
- [25] Z. NEHARI, *Characteristic values associated with a class of non-linear second order differential equations*, Acta Math. **105**, (1961), 141–175.
- [26] M. CONTI, E. M. MARCHINI, P. VITTORINO, *A well posedness result for nonlinear viscoelastic equations with memory*, Nonlinear Anal. Theory Methods Appl. **96**, (2014), 206–216.