

EXISTENCE AND GLOBAL BEHAVIOR OF POSITIVE SOLUTIONS OF SEMILINEAR FRACTIONAL DIRICHLET PROBLEMS IN EXTERIOR DOMAINS

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Abstract. In this paper, we establish the existence and the global asymptotic behavior of positive solutions in an exterior domain $\Omega \subset \mathbb{R}^d$, $d \geq 3$,

$$\begin{cases} (-\Delta)^{\frac{\alpha}{2}} x = f(t)x^p, & \text{in } \Omega, \\ x > 0, & \text{in } \Omega, \\ \lim_{t \rightarrow \partial\Omega} \delta(t)^{1-\frac{\alpha}{2}} x(t) = 0, \\ \lim_{|t| \rightarrow \infty} x(t) = 0, \end{cases}$$

where $(-\Delta)^{\frac{\alpha}{2}}$ is the infinitesimal generator of a killed symmetric α -stable process X^Ω on Ω , $0 < \alpha < 2$, $p < 1$ and the function f is positive and satisfies the suitable conditions related to the Karamata classes \mathcal{K}_0 and \mathcal{K}_∞ . Our approach relies on potential theory, Karamata regular variation theory, and the Schauder fixed point theorem.

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