Differential
Equations
& Applications
Volume 18, Number 1 (2026), 27–43
doi:10.7153/dea-2026-18-03

BOUNDED VARIATION SOLUTION TO A NONLINEAR 1–LAPLACIAN TYPE PROBLEM WITHOUT THE AMBROSETTI–RABINOWITZ CONDITION

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Abstract. In this paper, we investigate the existence of a bounded variation solution for the following 1-Laplacian equation

$$\begin{cases} -\Delta_1 u = \lambda f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where λ is a real parameter, $\Omega \subset \mathbb{R}^N$ $(N \geqslant 2)$ is a bounded domain with Lipschitz boundary, $\Delta_1 u = \operatorname{div}(\frac{Du}{|Du|})$ is the 1-Laplacian, and the function $f(x,u) \in C^0(\overline{\Omega} \times \mathbb{R}, \mathbb{R})$ is superlinear in u at infinity and satisfies a subcritical growth condition. We prove that under suitable conditions, for all $\lambda > 0$, the problem has at least one nontrivial solution without the Ambrosetti–Rabinowitz condition. The approach is based on an analysis of the associated p-Laplacian problem, followed by a thorough analysis of the asymptotic behavior of such solutions as $p \to 1^+$.

Mathematics subject classification (2020): 35J92, 35J60, 49J45.
Keywords and phrases: 1-Laplacian operator, variational methods, bounded variation.

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