Differential
Equations
& Applications
Volume 18, Number 2 (2026), 45–58
doi:10.7153/dea-2026-18-04

A NOTE ON THE BOUNDEDNESS OF VARIABLE HARDY OPERATORS ON WEIGHTED HERZ—MORREY SPACES WITH VARIABLE EXPONENTS

MEHVISH SULTAN, BABAR SULTAN* AND RENÉ ERLIN CASTILLO

Abstract. In this paper, we will introduce the idea of weighted Herz–Morrey spaces with variable exponents. Then the fractional Hardy-type operators of variable order is shown to be bounded in these spaces.

Mathematics subject classification (2020): 46E30, 47B38.
Keywords and phrases: Lebesgue spaces, weighted estimates, Hardy operators, weighted variable Herz-Morrey spaces.

REFERENCES

[1] L. DIENING AND A. NEKVINDA, *Muckenhoupt weights in variable exponent spaces*, Math. Inequal. Appl. **11** (2008), no. 2, 235–245.
[2] A. HUSSAIN, I. KHAN AND A. MOHAMED, *Variable Herz-Morrey estimates for rough fractional Hausdorff operator*, J. Inequal. Appl. **33** (2024).
[3] M. IZUKI, *Boundedness of sublinear operators on Herz spaces with variable exponent and application to wavelet characterization*, Analysis Mathematica **36** (2010), no. 1, 33–50.
[4] M. IZUKI, *Boundedness of vector-valued sublinear operators on Herz–Morrey spaces with variable exponent*, Mathematical Sciences Research Journal **13** (2009), no. 10, 243–253.
[5] M. IZUKI AND T. NOI, *An intrinsic square function on weighted Herz spaces with variable exponents*, J. Math. Inequal. **11** (2017), no. 3, 799–816.
[6] M. IZUKI AND T. NOI, *Boundedness of fractional integrals on weighted Herz spaces with variable exponent*, J. Inequal. Appl. (2016), Article ID 1142, 1–15.
[7] V. KOKILASHVILI, A. MESKHI, H. RAFEIRO AND S. SAMKO, *Integral Operators in Non-Standard Function Spaces. Vol. 1: Variable Exponent Lebesgue and Amalgam Spaces*, Oper. Theory Adv. Appl. **248**, Birkhäuser/Springer, Cham, 2016.
[8] V. KOKILASHVILI, A. MESKHI, H. RAFEIRO AND S. SAMKO, *Integral Operators in Non-Standard Function Spaces. Vol. 2: Variable Exponent Hölder, Morrey–Campanato and Grand Spaces*, Oper. Theory Adv. Appl. **249**, Birkhäuser/Springer, Cham, 2016.
[9] V. KOKILASHVILI AND S. SAMKO, *On Sobolev theorem for Riesz-type potentials in the Lebesgue spaces with variable exponent*, Z. Anal. Anwend. **22** (2003), 899–910.
[10] A. MESKHI, *Maximal functions, potentials and singular integrals in grand Morrey spaces*, Complex Var. Elliptic Equ. **55** (2010), 1007–1019.
[11] B. SULTAN, *Atomic decomposition of anisotropic grand variable weighted Herz spaces and applications*, J. Pseudo-Differ. Oper. Appl. **16** (2025), 53.
[12] B. SULTAN, *Atomic decomposition on grand Herz–Hardy spaces on locally compact Vilenkin groups*, Filomat **40** (2026), no. 4, 1301–1309.
[13] B. SULTAN, F. AZMI, M. SULTAN, M. MEHMOOD AND N. MLAIKI, *Boundedness of Riesz potential operator on grand Herz–Morrey spaces*, Axioms **11** (2022), no. 11, 583.
[14] B. SULTAN, F. AZMI, M. SULTAN, T. MAHMOOD, N. MLAIKI AND N. SOUAYAH, *Boundedness of fractional integrals on grand weighted Herz–Morrey spaces with variable exponent*, Fractal Fract. **6** (2022), no. 11, 660–670.

© ELEMENT, Zagreb
Paper DEA-18-04

[15] B. SULTAN AND A. HUSSAIN, *Sobolev embeddings in grand variable Herz-Morrey Besov spaces*, Bound. Value Probl. **2026** (2026), Article 11.
[16] B. SULTAN, A. HUSSAIN AND M. SULTAN, *Characterization of generalized Campanato spaces with variable exponents via fractional integrals*, J. Pseudo-Differ. Oper. Appl. **16** (2025), 22.
[17] B. SULTAN AND M. SULTAN, *Boundedness of commutators of rough Hardy operators on grand variable Herz spaces*, Forum Math. **36** (2024), no. 3, 717–733.
[18] B. SULTAN AND M. SULTAN, *Boundedness of higher order commutators of fractional variable rough Hardy operators on grand variable Herz spaces*, J. Classical Anal. **26** (2025), no. 1, 1–19.
[19] B. SULTAN AND M. SULTAN, *Boundedness of higher order commutators of Hardy operators on grand Herz–Morrey spaces*, Bull. Sci. Math. **190** (2024), 103373.
[20] B. SULTAN AND M. SULTAN, *Sobolev-type theorem for commutators of Hardy operators in grand Herz spaces*, Ukr. Math. J. (2024).
[21] B. SULTAN, M. SULTAN AND A. HUSSAIN, *Boundedness of the Bochner–Riesz operators on the weighted Herz–Morrey type Hardy spaces*, Complex Anal. Oper. Theory **19** (2025), 49.
[22] B. SULTAN, M. SULTAN AND I. KHAN, *On Sobolev theorem for higher commutators of fractional integrals in grand variable Herz spaces*, Commun. Nonlinear Sci. Numer. Simul. **126** (2023).
[23] B. SULTAN, M. SULTAN, M. MEHMOOD, F. AZMI, M. A. ALGHAFLI AND N. MLAIKI, *Boundedness of fractional integrals on grand weighted Herz spaces with variable exponent*, AIMS Math. **8** (2023), no. 1, 752–764.
[24] B. SULTAN, M. SULTAN, Q. Q. ZHANG, N. MLAIKI, *Boundedness of Hardy operators on grand variable weighted Herz spaces*, AIMS Math. **8** (2023), no. 10, 24515–24527.
[25] M. SULTAN AND B. SULTAN, *A note on the boundedness of higher order commutators on fractional integrals in grand variable Herz–Morrey spaces*, Kragujevac J. Math. **50** (2026), no. 7, 1063–1080.
[26] M. SULTAN AND B. SULTAN, *A note on the boundedness of the Marcinkiewicz integral operator on continual Herz–Morrey spaces*, Filomat **39** (2025), no. 6, 2017–2027.
[27] M. SULTAN AND B. SULTAN, *Boundedness of sublinear operators on grand central Orlicz–Morrey spaces*, Bull. Sci. Math. **205** (2025), 103704.
[28] M. SULTAN AND B. SULTAN, *λ-Central Musielak–Orlicz–Morrey spaces*, Arab. J. Math. **14** (2025), 357–363.
[29] M. SULTAN, B. SULTAN, A. ALOQAILY AND N. MLAIKI, *Boundedness of some operators on grand Herz spaces with variable exponent*, AIMS Math. **8** (2023), no. 6, 12964–12985.
[30] M. SULTAN, B. SULTAN AND R. E. CASTILLO, *Weighted composition operator on Gamma spaces with variable exponent*, J. Pseudo-Differ. Oper. Appl. **15** (2024), 46.
[31] L. WANG, *Parametrized Littlewood–Paley operators on grand variable Herz spaces*, Ann. Funct. Anal. **13** (2022), 72.
[32] J. L. WU AND W. J. ZHAO, *Boundedness for fractional Hardy-type operator on variable-exponent Herz–Morrey spaces*, Kyoto J. Math. **56** (2016), no. 4, 831–845.
[33] J. YOUNAS, A. HUSSAIN, H. ALHAZMI, A. F. ALJOHANI, I. KHAN, BMO *estimates for commutators of the rough fractional Hausdorff operator on grand-variable-Herz-Morrey spaces*, AIMS Math. **9** (2024), no. 9, 23434–23448.