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BOUNDARY VALUE PROBLEMS FOR NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS AND INCLUSIONS WITH NONLOCAL AND INTEGRAL BOUNDARY CONDITIONS

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Abstract. In this paper, we study a class of boundary value problems of nonlinear fractional differential equations and inclusions with nonlocal and integral boundary conditions. Some new existence and uniqueness results are obtained by using a variety of fixed point theorems. Examples are given to illustrate the results.

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REFERENCES

[1] R. P. AGARWAL, B. ANDRADE, C. CUEVAS, *Weighted pseudo-almost periodic solutions of a class of semilinear fractional differential equations*, Nonlinear Anal. Real World Appl. **11** (2010), 3532–3554.
[2] R.P. AGARWAL, V. LAKSHMIKANTHAM, J. J. NIETO, *On the concept of solution for fractional differential equations with uncertainty*, Nonlinear Anal. **72** (2010), 2859–2862.
[3] R.P. AGARWAL, Y. ZHOU, Y. HE, *Existence of fractional neutral functional differential equations*, Comput. Math. Appl. **59** (2010), 1095–1100.
[4] B. AHMAD, S. SIVASUNDARAM, *On four-point nonlocal boundary value problems of nonlinear integro-differential equations of fractional order*, Appl. Math. Comput. **217** (2010), 480–487.
[5] B. AHMAD, *Existence of solutions for irregular boundary value problems of nonlinear fractional differential equations*, Appl. Math. Lett. **23** (2010), 390–394.
[6] B. AHMAD, *Existence of solutions for fractional differential equations of order $q \in (2,3]$ with anti-periodic boundary conditions*, J. Appl. Math. Comput. **34** (2010), 385–391.
[7] B. AHMAD, *On nonlocal boundary value problems for nonlinear integro-differential equations of arbitrary fractional order*, Results Math. **63** (2013), 183–194.
[8] B. AHMAD, A. ALSAEDI, *Existence and uniqueness of solutions for coupled systems of higher order nonlinear fractional differential equations*, Fixed Point Theory Appl. **2010** (2010), Article ID 364560, 17 pp.
[9] B. AHMAD, S.K. NTOUYAS, A. ALSAEDI, *New existence results for nonlinear fractional differential equations with three-point integral boundary conditions*, Adv. Differ. Equ. **2011**, Article ID 107384, 11 pp.
[10] B. AHMAD, J.J. NIETO, *Existence results for nonlinear boundary value problems of fractional integrodifferential equations with integral boundary conditions*, Bound. Value Probl. **2009**, Art. ID 708576, 11 pp.
[11] B. AHMAD, J.J. NIETO, *Existence results for a coupled system of nonlinear fractional differential equations with three-point boundary conditions*, Comput. Math. Appl. **58** (2009), 1838–1843.
[12] Z.B. BAI, *On positive solutions of a nonlocal fractional boundary value problem*, Nonlinear Anal. **72** (2010), 916–924.
[13] K. BALACHANDRAN, J.J. TRUJILLO, *The nonlocal Cauchy problem for nonlinear fractional integrodifferential equations in Banach spaces*, Nonlinear Anal. **72** (2010), 4587–4593.
[14] M. BENCHOHRA, S. HAMANI, S.K. NTOUYAS, *Boundary value problems for differential equations with fractional order*, Surv. Math. Appl. **3** (2008), 1–12.

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[15] M. BENCHOHRA, S. HAMANI, S.K. NTOUYAS, *Boundary value problems for differential equations with fractional order and nonlocal conditions*, Nonlinear Anal. **71** (2009), 2391–2396.
[16] A.V. BITSADZE, *On the Theory of Nonlocal Boundary Value Problems*, Dokl. Akad. Nauk SSSR **277** (1984), 17–19.
[17] A. BRESSAN, G. COLOMBO, *Extensions and selections of maps with decomposable values*, Studia Math. **90** (1988), 69–86.
[18] L. BYSZEWSKI, V.LAKSHMIKANTHAM, *Theorem about the existence and uniqueness of a solution of a nonlocal abstract Cauchy problem in a Banach space*, Appl. Anal. **40** (1991), 11–19.
[19] L. BYSZEWSKI, *Theorems about existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem*, J. Math. Anal. Appl. **162** (1991), 494–505.
[20] L. BYSZEWSKI, *Existence and uniqueness of mild and classical solutions of semilinear functional-differential evolution nonlocal Cauchy problem*, Selected problems of mathematics, 50th Anniv. Cracow Univ. Technol. Anniv., Issue 6, Cracow Univ. Technol., Krakow, 1995, pp. 25-33.
[21] K. DEIMLING, *Multivalued Differential Equations*, Walter De Gruyter, Berlin-New York, 1992.
[22] M. FRIGON, *Théorèmes d'existence de solutions d'inclusions différentielles*, Topological Methods in Differential Equations and Inclusions (edited by A. Granas and M. Frigon), NATO ASI Series C, Vol. **472**, Kluwer Acad. Publ., Dordrecht, 1995, 51–87.
[23] SH. HU, N. PAPAGEORGIOU, *Handbook of Multivalued Analysis, Theory I*, Kluwer, Dordrecht, 1997.
[24] A. GRANAS AND J. DUGUNDJI, *Fixed Point Theory*, Springer-Verlag, New York, 2005.
[25] A.A. KILBAS, H.M. SRIVASTAVA, J.J. TRUJILLO, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, 204. Elsevier Science B.V., Amsterdam, 2006.
[26] A. LASOTA AND Z. OPIAL, *An application of the Kakutani-Ky Fan theorem in the theory of ordinary differential equations*, Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys. **13** (1965), 781–786.
[27] M.P. LAZAREVIĆ, A.M. SPASIĆ, *Finite-time stability analysis of fractional order time-delay systems: Gronwall's approach*, Math. Comput. Model. **49** (2009), 475–481.
[28] J.J. NIETO, *Maximum principles for fractional differential equations derived from Mittag-Leffler functions*, Appl. Math. Lett. **23** (2010), 1248–1251.
[29] D. O'REGAN, *Fixed-point theory for the sum of two operators*, Appl. Math. Lett. **9** (1996), 1–8.
[30] W. V. PETRYSHYN AND P.M. FITZPATRIC, *A degree theory, fixed point theoremsand mapping theorems for multivalued noncompact maps*, Trans. Amer. Math. Soc. **194** (1974), 1–25.
[31] I. PODLUBNY, *Fractional Differential Equations*, Academic Press, San Diego, 1999.
[32] J. SABATIER, O.P. AGRAWAL, J.A.T. MACHADO (Eds.), *Advances in Fractional Calculus: Theoretical Developments and Applications in Physics and Engineering*, Springer, Dordrecht, 2007.
[33] S. G. SAMKO, A.A. KILBAS, O.I. MARICHEV, *Fractional Integrals and Derivatives, Theory and Applications*, Gordon and Breach, Yverdon, 1993.
[34] Z. WEI, Q. LI, J. CHE, *Initial value problems for fractional differential equations involving RiemannLiouville sequential fractional derivative*, J. Math. Anal. Appl. **367** (2010), 260–272.
[35] S. Q. ZHANG, *Positive solutions to singular boundary value problem for nonlinear fractional differential equation*, Comput. Math. Appl. **59** (2010), 1300–1309.
[36] W. ZHONG, W. LIN, *Nonlocal and multiple-point boundary value problem for fractional differential equations*, Comput. Math. Appl. **39** (2010), 1345–1351.