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EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR FRACTIONAL ORDER m–POINT BOUNDARY VALUE PROBLEMS

KAMAL SHAH, SALMAN ZEB AND RAHMAT ALI KHAN

Abstract. This paper deals with the existence of solutions for a class of fractional order differential equations having m-points boundary conditions involving the Caputo fractional derivative. Moreover the nonlinearity also depend on the Caputo fractional derivative. We obtain sufficient conditions for the existence and uniqueness of solutions via Schauder's fixed-point theorem and Banach contraction principle. We provide an example to illustrate the applicability of our results.

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