

## A NONLOCAL FRACTIONAL HELMHOLTZ EQUATION

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**Abstract.** In this paper we study some boundary value problems for a fractional analogue of second order elliptic equation with an involution perturbation in a rectangular domain. Theorems on existence and uniqueness of a solution of the considered problems are proved by spectral method.

**Mathematics subject classification (2010):** 35R30, 35K05, 35K20.

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