

## SUFFICIENT CONDITIONS FOR THE EXPONENTIAL STABILITY OF NONLINEAR FRACTIONALLY PERTURBED ODEs WITH MULTIPLE TIME DELAYS

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**Abstract.** In this paper we prove some results on the exponential stability of the trivial solution of a system of fractional differential equations with multiple delays and tempered Riemann-Liouville fractional integrals on its right-hand side.

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